Re: equilateral tringles

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- *From*: rusin@xxxxxxxxxxxxxxxxxxxxx (Dave Rusin)
- *Date*: 5 Dec 2005 20:17:35 GMT

In article <3950172.1133377985893.JavaMail.jakarta@xxxxxxxxxxxxxxxxxxxxxx>,
eugene <jane1806@xxxxxxx> wrote:
> Let ABC be an equilateral triangle. Points A_1,B_1,C_1 are chosen
> inside the triangle in such a way that A_1 \in CC_1, B_1 \in AA_1,
> C_1 \in BB_1 and AB_1=B_1A_1, BC_1=C_1B_1, CA_1=C_1A_1. Prove that
> the triangle A_1B_1C_1 is also equilateral.

I "calculated" the locations of A1 B1 C1, or at least, I showed that
the stipulations lead to a unique A1, B1, C1; then from the symmetry
of the problem it follows A1 B1 C1 form an equilateral triangle.

Sensing objections, I gave another problem to put this into context:

In article <dmlre9tqk1@xxxxxxxxxxxxxxxxx>,
Dave Rusin <rusin@xxxxxxxxxxxxxxxxxxxxx> wrote:

>Here is a variant: What happens if we try this trick starting with
>a regular tetrahedron P_1 P_2 P_3 P_4 ? Construct four more
>points Q_i with Q_{i+1} being the midpoints of Q_i P_i.
>Is there a unique such set of four points? Do they form a regular
>tetrahedron? How does this shed light on what happened in the plane?

No one took the bait so let me explain.

You can in fact carry out this construction (e.g. using again either
barycentric coordinates, or an embedding of the tetrahedron into R^3).
Again the solution is unique, and we can compute the coordinates.
Again we can appeal to symmetry BUT this time we note carefully that
the symmetry of the problem is only C_4 −− a circular permutation
of the four vertices produces an identical problem, and therefore
the underlying rotations preserve the polyhedron Q1 Q2 Q3 Q4 setwise.

But that's not enough to make a tetrahedron! Tetrahedra are invariant
(setwise) under the linear isometries induced by ANY permutation of
the four vertices, and C_4 is a long way from S_4. In fact, if
you compute the distance between Q1 and Q3 you'll see it's
different from the distance from Q1 to Q2 or Q4.

So what, really, makes the triangle equilateral in the planar case?
Again we have C_3 symmetry by the symmetry + uniqueness of the
construction. In particular, any pair $\{Q_i, Q_j\}$ can be rotated
to any other pair $\{Q_k, Q_l\}$ SETWISE (but not necessarily
preserving orderings within the pairs). However, since a metric is
symmetric, this means that all distances $d(Q_i, Q_j)$ are equal
(as long as $i <> j$ of course), and that makes the triangle
equilateral. So we have a little assist from symmetry of the metric
to move from $C_3 = A_3$ symmetry of the solution to S_3 symmetry.

The tetrahedral case gets the same boost, of course: an A_4−symmetric
tetrahedron is also S_4−symmetric (i.e. regular). But this time
C_4 and A_4 are distinct (indeed, we don't even have C_4 contained
in A_4).

I hope that clarifies just a little why the triangle is equilateral.

dave
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- ***References***:
  - ***equilateral tringles***
    - *From:* eugene
  - ***Re: equilateral tringles***
    - *From:* Dave Rusin
  - ***Re: equilateral tringles***
    - *From:* Philippe 92
  - ***Re: equilateral tringles***
    - *From:* Dave Rusin

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