

Re: Can I have fries and a calculator with that?

Source: <http://sci.tech-archive.net/Archive/sci.math/2005-12/msg02203.html>

- *From:* "Peter Webb" <webbfamily-diespamdie@xxxxxxxxxxxxxxxxxxx>
 - *Date:* Sun, 11 Dec 2005 02:08:46 +1100
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"Bill Dubuque" <wgd@xxxxxxxxxxxxxxxxxxxxxxxx> wrote in message news:y8zwtid4b0u.fsf@xxxxxxxxxxxxxxxxxxxxxxxx
> "Dave L. Renfro" <renfr1dl@xxxxxxxx> wrote:
>>
>> As for rationalizing denominators, here's a less trivial
>> example for you. We can rationalize the denominator of $1/b$,
>>
>> where $b = \sqrt{2} + \sqrt{10} + \sqrt{12} + \sqrt{56}$,
>>
>> by multiplying the numerator and denominator by $f(b)$, where
>>
>> $f(x) = x^{15} - (640)x^{13} + (155,072)x^{11} - (18,296,832)x^9$
>>
>> $+ (1,125,983,744)x^7 - (35,305,193,472)x^5$
>>
>> $+ (491,646,992,384)x^3 - (1,840,594,812,928)x$.
>>
>> After multiplying, but before reducing, the denominator will be
>>
>> $d = -525,242,269,696$.
>>
>> The numerator will be an integer-linear combination of 15
>> rationally independent square root terms.
>
> I.e. $g(x) := x f(x) - d$ is the minimum polynomial of b over \mathbb{Q} .
> Of course one may employ the minimum poly of any algebraic number
> w in a similar manner to rationalize w in a denominator. This is
> essentially the same as multiplying w by all of its conjugates.

Indulge me here a little, this is a hobby (not a good start, I know).

We know that $\sin(90 \text{ degrees}) = 1$, and we know that the value of $\sin(45 \text{ degrees})$ satisfies the equation $2x^2 = 1$.

We further know that the value of $\sin(1 \text{ degree})$ satisfies some equation $\sum (\text{choose}(90,i)x^i) = 1$ (or similar).

Therefore $\sin(1 \text{ degree})$ is algebraic, and so is $\sin(1+1)$.

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We can therefore construct $\sin(n)$ or indeed $\sin(m/n)$ as algebraic.

Therefore \sin maps all rational degrees to algebraic numbers.

The interesting (for me) is the fact that this doesn't occur for \sin in radians, where $\sin(1)$ seems unobtainable within algebraics.

- *Follow-Ups:*

- ◆ *Re: Can I have fries and a calculator with that?*

◇ *From:* Dave L. Renfro

- *References:*

- ◆ *Can I have fries and a calculator with that?*

◇ *From:* Dave L. Renfro

- ◆ *Re: Can I have fries and a calculator with that?*

◇ *From:* Virgil

- ◆ *Re: Can I have fries and a calculator with that?*

◇ *From:* Dave L. Renfro

- ◆ *Re: Can I have fries and a calculator with that?*

◇ *From:* Bill Dubuque

- Prev by Date: *Re: How to prove that $\exp(x)$ is not uniform convergent in \mathbb{R} ?*

- Next by Date: *Re: recurrent sequence*

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