what "REALLY" is derivative?

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Hi,

Back in grade 11 when learning about parabolas, a teacher told me that there are lines that can touch the parabola only at one point, for every point on the curve (tangents), and that we needed to wait for more advanced courses to find the equation of these lines.

I thought this was nonsense, because if two curves (parabola and line) intersect, then it's easy to solve for the slope of the line (because we knew how to solve the intersection of two lines− "this can't be much different" i thougth).. this is what I did:

equation of parabola: y = ax^2 + bx + c

point slope equation of line: y − y1 = m(x − x1)

what I did was substitute the point (q , aq^2 + bq + c) for (x1,y1) in the slope equation

y − aq^2 − bq − c = m(x − q)

then substitute the y = aq^2 + bq + c

ax^2 + bx + c − aq^2 − bq − c = m(x − q)

a(x^2 − q^2) + b(x − q)= m(x − q)

(x−q) [a(x+q) + b] = m(x − q)

THEN I DID THE UNTHINKABLE and divided both sides by (x−q)

m = a(x+q) + b and that is the slope of a line touching parabola at x = q

I had no clue what a limit was, or a derivative, but I used the ideas learned about the intersection of two lines, and used it for the intersection of a parbola and a line (and just ASSUMED that x−q != 0)

I then tried this for other curves successfully, and when I started learning calculus (2 years later), I couldn't get past the fact that the limit seemed to me (for reasons shown above) as an excuse to allow us to divide f(x) − f(a) by x − a.

How is there always a x−a (for us to cancel out) within f(x) − f(a) if f is differentiable at 'a'. Does anyone know how this really works? Have any of you wondered how solving limits (be in with e/d or using limit laws/continuity without e/d) always works out perfect (we always get a factor x−a or h that cancels out or we always get the magical absolute value term with e−d limit proofs)? I know if f(x) − f(a) is a polynomial, it is zero when x = a, so there must be a factor x−a (by factor theorem) ... but really.. how "really" does this work?

My post sounds unintelligible because I can't shape my confusions/thoughts properly to explain it clearly (and English is not my native tongue). My "funky" grade 11 self discovered method is clashing with the limit of difference quotient definition of a derivative. My extreme inquisitiveness (yes extreme, which is why I always do bad in school, because I can't get over the "WHYYYYY?" even if I"m told the WHYYY?? is coming later on).

Can anyone shed some light on my confusion? What "really" is a derivative?

Any insights or guidance would be greatly appreciated.

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