Re: what "REALLY" is derivative?

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- *From*: mskirvin@xxxxxxxxx

- *Date*: 13 Dec 2005 13:47:20 −0800

Luke Wu wrote:
> Hi,
>
> Back in grade 11 when learning about parabolas, a teacher told me that
> there are lines that can touch the parabola only at one point, for
> every point on the curve (tangents), and that we needed to wait for
> more advanced courses to find the equation of these lines.
>
> I thought this was nonsense, because if two curves (parabola and line)
> intersect, then it's easy to solve for the slope of the line (because
> we knew how to solve the intersection of two lines− "this can't be much
> different" i thougth).. this is what I did:
>
> equation of parabola: y = ax^2 + bx + c
>
> point slope equation of line: y − y1 = m(x − x1)
>
> what I did was substitute the point (q , aq^2 + bq + c) for (x1,y1) in
> the slope equation
>
>
> y − aq^2 − bq − c = m(x − q)
>
> then substitute the y = aq^2 + bq + c
>
> ax^2 + bx + c − aq^2 − bq − c = m(x − q)
>
> a(x^2 − q^2) + b(x − q)= m(x − q)
>
> (x−q) [a(x+q) + b] = m(x − q)
>
> THEN I DID THE UNTHINKABLE and divided both sides by (x−q)
>
> m = a(x+q) + b and that is the slope of a line touching parabola at x
> = q
>
> I had no clue what a limit was, or a derivative, but I used the ideas
> learned about the intersection of two lines, and used it for the

> intersection of a parbola and a line (and just ASSUMED that x−q != 0)
>
> I then tried this for other curves successfully, and when I started
> learning calculus (2 years later), I couldn't get past the fact that
> the limit seemed to me (for reasons shown above) as an excuse to allow
> us to divide f(x) − f(a) by x − a.
>
> How is there always a x−a (for us to cancel out) within f(x) − f(a) if
> f is differentiable at 'a'. Does anyone know how this really works?
> Have any of you wondered how solving limits (be in with e/d or using
> limit laws/continuity without e/d) always works out perfect (we always
> get a factor x−a or h that cancels out or we always get the magical
> absolute value term with e−d limit proofs)? I know if f(x) − f(a) is a
> polynomial, it is zero when x = a, so there must be a factor x−a (by
> factor theorem) ... but really.. how "really" does this work?
>
> My post sounds unintelligible because I can't shape my
> confusions/thoughts properly to explain it clearly (and English is not
> my native tongue). My "funky" grade 11 self discovered met