

# Re: Insphere of a tetrahedron

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*Source:* <http://sci.tech-archive.net/Archive/sci.math/2006-01/msg02897.html>

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  - *Date:* Thu, 19 Jan 2006 23:14:22 +0000 (UTC)
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In article <dqid7f\$7iv\$2@xxxxxxxxxxxxxxxxxxxxxxxxxxxx>, Hauke Reddmann <fc3a501@xxxxxxxxxxxxxxxx> wrote:  
>Given an irregular (!!) tetrahedron as 4 space point coordinates.  
>What are the coordinates of the insphere center?  
>(Paper reference suffices :-)  
>(Tried to compute that myself, straightforward but the formula  
>piggifies exponentially so a trick would help.)

OK, I worked this out. The results are so simple that I can't help but to assume that they must be known.

Theorem 1:

Let  $a, b, c$  be the lengths of the sides opposite to vertices  $A, B, C$  of the triangle  $ABC$  and let  $O$  be the triangle's incenter. Then we have:  
$$O = (a/t) A + (b/t) B + (c/t) C,$$
where  $t = a + b + c$  is the triangle's perimeter.

Note: Here I am viewing  $A, B, C, O$  as points in  $R^2$ . The formula above gives the incenter  $O$  as a convex combination of the vertices  $A, B, C$ .

Theorem 2:

Let  $a, b, c, d$  be the areas of the faces opposite to vertices  $A, B, C, D$  of the tetrahedron  $ABCD$  and let  $O$  be the tetrahedron's incenter. Then we have:  
$$O = (a/t) A + (b/t) B + (c/t) C + (d/t) D,$$
where  $t = a + b + c + d$  is the tetrahedron's surface area.

This, too, gives  $O$  as a convex combination of the vertices. It is likely that the formula generalizes to higher dimensions in the obvious way but I haven't given much thought to that.

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Rouben Rostamian

Re: Insphere of a tetrahedron

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