Re: Existence of pdf and mgf

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- *From*: israel@xxxxxxxxxxx (Robert Israel)
- *Date*: 27 Jan 2006 20:03:23 GMT

In article <43da3f99$1@xxxxxxxxxxxxxxxxx>,
Olw <anders_REMOVE@xxxxxxxxxxx> wrote:

>I have some struggles understanding existence implications between the
>moment generating function and a probability distribution.
>
>Going from a pdf / cdf to an MGF is OK, the MGF M(t)exists if the
>expectation E(e^{tx}) exist at some neighborhood of 0.
>
>The other way, however. Given a function claiming to be an MGF, for
>example M(t) = t or M(t)=t/(1−t), for |t|<1. How can one know if it
>exists a corresponding pdf?

The MGF for imaginary t is the characteristic function
phi(t) = E[exp(itX)]. According to Bochner's theorem, a
function phi(t) is the characteristic function of a random
variable [in analyst's language, the Fourier transform of
a Borel probability measure on the real line] if and only if
it is continuous, positive definite, and phi(0) = 1.
Positive definite means for all real t_1, ..., t_n, the
n x n matrix with entries a_{j,k} = phi(t_j − t_k) is positive
semidefinite.

>Also, say that one can decide that an MGF is valid, i.e., there is a
>corresponding pdf. Then the pdf can be found by inverse Laplace
>transforming M(−t). Is this in convenient also for distribution that can
>take negative values? Or does it exist any clever tricks for finding the
>pdf?

Fourier transform of the characteristic function.

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