

## Re: axiom of choice and equivalent statements

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  - *Date:* Sun, 05 Feb 2006 15:58:53 GMT
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"Li Yi" <[liyi.cn@xxxxxxxxxx](mailto:liyi.cn@xxxxxxxxxx)> wrote in message  
<news:1139145297.879035.68570@xx>

Prove the following statements are equivalent:

- (1) axiom of choice;
- (2) for any sets A and B, it holds either  $A \leq B$  or  $B \leq A$ . ( $X \leq Y$  means that there is an injection from X to Y)
- (3) Zorn's lemma
- (4) Well-ordering Principle : every set can be well-ordered.
- (5) Every binary relation can be single-valued: Given a binary relation R, there exists a function  $F \subseteq R$  such that  $\text{dom } F = \text{dom } R$
- (6) A is a nonempty set, then  $A^a$  is nonempty for any ordinal number a.

I know that (1)(3)(4) are equivalent and have a proof of (3) $\Rightarrow$ (2).

The simplest solution to your problem is to get from your school library a copy of "Equivalents of the Axiom of Choice". First edition is Rubin & Rubin. Second addition added someone else whose name I can't recall.

Norm

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