Re: summary of {1/n}

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- *From*: rob@xxxxxxxxxxxxxx (Rob Johnson)
- *Date*: Mon, 06 Feb 2006 15:57:38 GMT

In article <1139237115.327293.36150@xxxxxxxxxxxxxxxxxxxxxxxxxxxxx>,
"Steven Woody" <narkewoody@xxxxxxxxx> wrote:

> what's the solution of summary of {1/n} ? thanks.

I'm guessing that by summary, you mean sum or summation. However,
I can't guess whether you mean the sum from n=1 to infinity or from
n=1 to some finite N. Do you want the exact formula, which is not
as useful for numerical purposes, or an asymptotic expansion, which
is not exact, but better for numerical purposes.

The complete series diverges, that is

oo
−−− 1
> − = oo
−−− n
n=1

The exact sum is

N
−−− 1 Gamma'(N)
> − = gamma + −−−−−−−−−
−−− n Gamma(N)
n=1

where gamma is the Euler−Mascheroni constant

<http://mathworld.wolfram.com/Euler−MascheroniConstant.html>

and Gamma is the Gamma function

<http://mathworld.wolfram.com/GammaFunction.html>

The asymptotic expansion is

N

−−− 1 1 1 1 1 1 1
> − ~ log(N) + gamma + −− − −−−−− + −−−−−− − −−−−−− + −−−−−− − −−−−−−− + ...
−−− n 2N 12N^2 120N^4 252N^6 240N^8 132N^10
n=1

See <http://www.whim.org/nebula/math/harmonic.html>.

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