Re: Index 2 subgroups of Lie groups

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In article <dstvij$25i9$1@xxxxxxxxxxxxxxxxx>, James
<James545@xxxxxxxxx> wrote:

> Hi,
>
> I know SO(2) is an index 2 subgroup of O(2). Are there any other Lie groups
> that have index 2 subgroups? Is SO(n) also an index 2 subgroup of O(n)? (I
> know O(2) is generated by rotations and reflections, and SO(2) is generated
> by rotations, which is where we get index 2, but is this true in general for
> O(n) and SO(n)?)
> What about other Lie groups with index 2 subgroups other than orthogonal
> groups?
>
> Thank you,

If G is any Lie group, the G x (Z/2Z) is a Lie group containing (a copy
of) G as a subgroup of index 2.

More generally, if G has a continuous automorphism of period 2, then
there is a semidirect product of G with (Z/2Z), in which a copy of G is
the normal subgroup.

Hmm... are there any other cases? (I thought this would be an easy
exercise with the answer "No"... but on second thought I'm not at all
sure.)

−−
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God just doesn't fit inside a single religion.
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