# How can the meaning of Goedel's unprovable statement descend from infinity?

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(Note: The author is aware that solutions of certain Diophantine equations are said to be mechanically uncheckable. This post focuses strictly on Goedel's Incompleteness Theorem, as it is proved by J.N. Crossley in "What Is Mathematical Logic?")

Don't you find it curious that the proof of Goedel's Incompleteness Theorem relies on "free variables"? When do these "free variables" ever appear in axioms or theorems in arithmetic? Why are they so important in proving the existence of a statement that cannot be proved or disproved?

I have thought hard about Goedel's Incompleteness Theorem since I read a proof at age fifteen. My goal was to intuitively understand the meaning of the unprovable statement. I now believe that while the theorem is true, the statement is meaningless. It is just not the kind of statement we would wish to prove in arithmetic. In this post, I will attempt not a disproof of Goedel's theorem itself, but a proof of the "meaninglessness" of the unprovable statement.

First, a little background information.

The symbols of arithmetic (+, −, =, logic symbols, etc.) may be assigned certain numbers, so that statements in arithmetic may be coded into a "Goedel number" by making the assigned numbers exponents of the primes, in the order they occur.

We can define what it means for a number to be the "Goedel number of a proof": we make the Goedel numbers of sequentially derived statements exponents of the primes, in the order they are deduced. It is a proof if each deduction is valid, and this can be checked mechanically.

Let "x sub y" be the Goedel number that results when all the free variables of the statement of the Goedel number x are replaced with the Goedel coding of the number y. In essence, x sub y refers to a function by which the value of y is "filled in" for the free variables in x.

Let Pf(x, y) be *not* the property that x is the Goedel number of a proof of Goedel number y, but the *Goedel number of the statement* that x is the Goedel number of a proof of the Goedel number y.

I give a brief outline of Crossley's proof here:

Let g be the Goedel number of the statement ~(Ex)(Pf(x, y sub y)). This is read, "There is no proof of y sub y." It contains one free variable, y, twice. Now consider the statement "g sub g." One can easily see from the definition of g that it is