Re: Challenae question for mathematician

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- *From*: "Arturo Magidin" <magidin@xxxxxxxxxxxxxxxxx>
- *Date*: 17 Feb 2006 12:05:26 −0800

C6L1V@xxxxxxx wrote:

> Arturo Magidin wrote:

[.snip.]

>> If we let S = { (1,2), (2,3), (3,4), ..., (n−1,n) }.
>>
>> We can express (a)(b)^{−1} = (3,4) as a product of elements of S by
>>
>> (3,4) = (3,4).
>>
>> You can express (c)(b)^{−1} = (3,5) as a product of elements of S by
>>
>> (3,5) = (3,5)(4,5)(3,4) [composing right to left]
>>
>> For an element x of S_n, define its S−length to be the least number of factors needed to express x as a product of elements of S.

> Just as a matter of interest, how does one determine the S−length, since there are many distinct product representations of the same S?

I confess that I am not particularly well−versed in algorithmic problems relating to permutation groups. I know that's a weak point in my proposal, if no good algorithmic way exists to do so.

Certainly you can do it in exponential time, since there is an easy way to express any cycle as products of elements of S, which gives you an easy upper bound for length_S(x) for any x (add the lengths of the cycles).

>> Define the

> > "distance" from x to y, d(x,y) to be the length of xy^{-1}.
>
> Is this the same as the minimum number of factors needed to get from x to y?

It would be: say you can get from x to y by applying s_1, s_2, s_3,.... s_k. That means that

$s_1*s_2*...*s_k x = y$

from which you get

$xy^{-1} = s_k^{-1} * ... * s_1^{-1} = s_k * ... * s_1$

so the number of transpositions needed to convert x into y is greater than or equal to d(x,y). And if d(x,y) = k, then $xy^{-1} = s_1 * ... * s_k$ for some s_i in S, from which you get
$s_k*...*s_1*x = y$, hence you need at most d(x,y) transpositions to get from x to y.

Arturo Magidin, sans .sig

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