

Re: A question about Caontor's proof of the uncountability of the reals

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On Mon, 27 Feb 2006 06:06:23 GMT, Michael Olea wrote:

This, I think, misconstrues my post. I have never had any trouble understanding Cantor's diagonalization proof: assume the reals are countable, show that this assumption leads to a contradiction, conclude that therefore that the assumption was false. This all makes perfect sense. It is only when the law of the excluded middle comes under fire – something I've only given any attention recently – that there may be room for seeds of doubt. Are there conditions under which (P) and (not P) are not the only possibilities, and therefore establishing that not P is false need not imply the P is true? So the real question was not about Cantor's proof of the uncountability of the reals per se, but about the validity of proof by contradiction in axiomatic systems where Godel's incompleteness theorems apply.

Are you aware that direct proofs exist?

Let  $f: \mathbb{N} \rightarrow \mathbb{R}$  be an arbitrary mapping. Show that there exists  $x$  in  $\mathbb{R}$  that is not in the range of  $f$ , and hence  $f$  is not a surjection. Since  $f$  is arbitrary, we conclude that no surjection exists.

This is a direct proof. There is no contradiction in sight, and no reliance on excluded middle. Does this answer your objection?

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Dave Seaman  
U.S. Court of Appeals to review three issues  
concerning case of Mumia Abu-Jamal.  
<<http://www.mumia2000.org/>>