Re: Integer−Valued Polynomials

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- *From*: "Chip Eastham" <hardmath@xxxxxxxxx>
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Maury Barbato wrote:

> Hello,
> let D be a domain, K its quotient field, and F an
> extension field of K. The ring of integer−valued
> polynomials in n indetermintes over D consists of those
> polynomials in K[x_1,...,x_n] which give a value in D
> for every argument in D^n (standard definition).
> What happens if we replace K[x_1,...,x_n] with
> F[x_1,...,x_n] in this definition? Do we obtain the same
> set of polynomials? What if D=Z, K=Q and F=C?
> Thank you very mauch for your ideas.

Hi, Maury:

I don't believe we do pick anything "extra" in polynomials
over the extension field K, by an interpolation argument.

First let us note that one can have a "integer−valued"
polynomial which is not simply integer coefficients:

f(x) = x(x+1)/2

for example. However a polynomial with coefficients in
(for example) the complex field C and integer−valued
for all arguments in Z must agree with a polynomial of
equal degree with coefficients in Q (e.g. constructed
by Newton divided differences) on all arguments Z,
and hence be identical to that polynomial over Q.

regards, chip

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