

# Re: Volume of a section of a sphere

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"Brian Blandford" <[b.blandford@xxxxxxxxxxxxxxxxxxxxxxxxxxxxx](mailto:b.blandford@xxxxxxxxxxxxxxxxxxxxxxxxxxxxx)> wrote in message [news:IH4Nf.70650\\$DM.31204@xxxxxxxxxxxxxxxxxxxxxxxxxxxxx](news:IH4Nf.70650$DM.31204@xxxxxxxxxxxxxxxxxxxxxxxxxxxxx)

A unit sphere is cut into 4 sections by two perpendicular planes at distances  $a, b$  from the centre.  $0 < a, b < 1$ . What is the volume of the smallest section?

Think of the problem in two dimensions and then generalize.

In two dimensions you are looking at the unit circle with two perpendicular lines cutting it at distances  $a$  and  $b$ .

Think of the intersection of the lines as your new origin. This amounts to displacing the circle so its center becomes  $(-a, -b)$ . Then your circle's equation with respect to the new origin will be  $(x+a)^2 + (y+b)^2 = 1$ .

You then want to find the area of the plane bounded by the translated circle and the positive  $x$  and  $y$  axes. The circle intersects the  $x$ -axis at the positive solution of  $(x+a)^2 + b^2 = 1$ .

Call this solution  $x_0$ . Then your area will be:

$$\int_0^{x_0} (\sqrt{1-(x+a)^2} - b) dx$$

Now generalize the above to three dimensions.

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