Symbolic Logic write−up of Infinitude of Primes, direct & indirect methods; Infinitude of Twin Primes

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a.p.:

> And it begs the question, how many assumptions can you make in a reductio ad absurdum before it is absurd. Suppose the Earth is flat. Suppose the Moon is green cheese. Suppose the weak nuclear force does not exist. Can anyone honestly think they could navigate in such a pile of crap?

You can do abbitary number of assumptions in a proof.
I will give an example where I proof that there
are infinitely many primes that leaves 2 as reaminder
upon division by 3.

Assumptions 1:
Suppose that there are fintitely many primes of the form 3N+2.

Let those prime bee p1,p2,p3..,pn

Form the number Q=p1^2.p2^2.p3^2..pn^2+1

Each of the p:s are of the form 3N+2.
The square of a number 3N+2 is of 3N+1
And a product of 3N+1 is of 3N+1
So Q is of 3N+2

ALL NUMBERS CAN BEE written as a product of primes.
NOTE: I don't assume the Unique Factorization Theorem.

Either Q is prime or it's composite.
Assumptions 1a:Q is prime.
Assumptions 1b:Q is composite.
Note: I don't do both these assumption at the same time.

First let it bee prime, then I have a prime of 3N+2
Bigger than all the p:s.

Second let it bee composite:

Q=q1.q2.q3..qm

p1^2.p2^2.p3^2..pn^2+1= q1.q2.q3..qm

Assumptions 2:
Suppose that one of the q:s is equal to one of the p:s.
Note: I make this assumption only if the first asumption is true.

Let for example p3=q4
Then one is divisible by that prime.

Therfore no of the q:s are equal to any of the p:s.

Assumptions 3:
Suppose that all the q:s is of 3N+1
Note: I make this assumption only if the first two asumptions is true.

Then Q is a product of 3N+1 and hence of 3N+1.
But Q is of 3N+2.
Hence some of the q:s is of 3N+2
Then i have a prime of 3N+2, not on the original list.

CONCLUSION:
THERE ARE INFINITELY MANY PRIMES OF THE FORM 3N+2.

DIAGRAM:
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/ \
a / \ b
Finitely many primes of 3N+2 Infinitely many primes of 3N+2
/ \
/ \
/ \
aa/ \ab
Q is prime Q is composite
/ \
/ \
/ \
aba/ \abb
one of the q:s equal No q equal to any p.
to one of the p:s / \
/ \
/ \
abba/ \abbb
all the q:s is of 3N+1 some q of 3N+2

either a or b is true
if a is true, either aa or ab is true

if ab is true, either aba or abb is true
if abb is true, either abba or abbb is true

hence:

either
1.a and aa
or
2.a and ab and aba
or
3.a and ab and abb and abba
or
4.a and ab and abb and abbb
or
5.b

IN case 1 I got a new prime and hence reductio ad absurdum to a.
IN case 2 I got one divisible by a prime, reductio ad absurdum.
IN case 3 I got a number OF 3N+2 as a product of 3N+1, reductio ad absurdum.
IN case 4 I got a prime of 3N+2 not on the original list, reductio ad absurdum to a.

Hence 1,2,3,4 is false and b is true.

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