

Symbolic Logic write-up of Infinitude of Primes, direct & indirect methods; Infinitude of Twin Primes

Source: <http://sci.tech-archive.net/Archive/sci.math/2006-03/msg01653.html>

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 - *Date:* Thu, 09 Mar 2006 22:18:17 GMT
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a.p.:

And it begs the question, how many assumptions can you make in a reductio ad absurdum before it is absurd. Suppose the Earth is flat. Suppose the Moon is green cheese. Suppose the weak nuclear force does not exist. Can anyone honestly think they could navigate in such a pile of crap?

You can do arbitrary number of assumptions in a proof. I will give an example where I prove that there are infinitely many primes that leaves 2 as remainder upon division by 3.

Assumptions 1:
Suppose that there are finitely many primes of the form $3N+2$.

Let those prime be $p_1, p_2, p_3, \dots, p_n$

Form the number $Q = p_1^2 \cdot p_2^2 \cdot p_3^2 \cdot \dots \cdot p_n^2 + 1$

Each of the p_i s are of the form $3N+2$.
The square of a number $3N+2$ is of $3N+1$
And a product of $3N+1$ is of $3N+1$
So Q is of $3N+2$

ALL NUMBERS CAN BE written as a product of primes.
NOTE: I don't assume the Unique Factorization Theorem.

Either Q is prime or it's composite.
Assumptions 1a: Q is prime.
Assumptions 1b: Q is composite.
Note: I don't do both these assumption at the same time.

First let it be prime, then I have a prime of $3N+2$
Bigger than all the p_i s.

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Second let it be composite:

$$Q = q_1 \cdot q_2 \cdot q_3 \dots q_m$$

$$p_1^2 \cdot p_2^2 \cdot p_3^2 \dots p_n^2 + 1 = q_1 \cdot q_2 \cdot q_3 \dots q_m$$

Assumptions 2:

Suppose that one of the q :s is equal to one of the p :s.

Note: I make this assumption only if the first assumption is true.

Let for example $p_3 = q_4$

Then one is divisible by that prime.

Therefore none of the q :s are equal to any of the p :s.

Assumptions 3:

Suppose that all the q :s is of $3N+1$

Note: I make this assumption only if the first two assumptions are true.

Then Q is a product of $3N+1$ and hence of $3N+1$.

But Q is of $3N+2$.

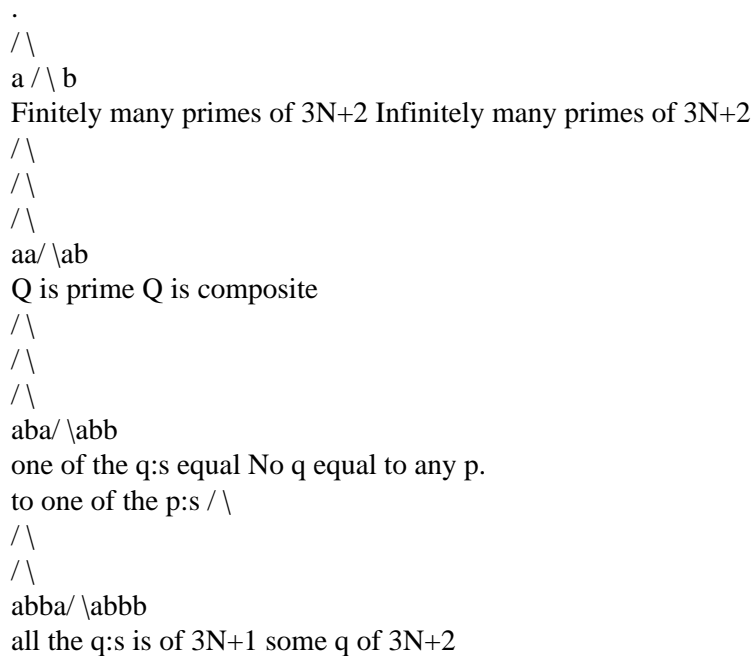
Hence some of the q :s is of $3N+2$

Then I have a prime of $3N+2$, not on the original list.

CONCLUSION:

THERE ARE INFINITELY MANY PRIMES OF THE FORM $3N+2$.

DIAGRAM:



either a or b is true

if a is true, either aa or ab is true

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if ab is true, either aba or abb is true

if abb is true, either $abba$ or $abbb$ is true

hence:

either

1. a and aa

or

2. a and ab and aba

or

3. a and ab and abb and $abba$

or

4. a and ab and abb and $abbb$

or

5. b

IN case 1 I got a new prime and hence reductio ad absurdum to a .

IN case 2 I got one divisible by a prime, reductio ad absurdum.

IN case 3 I got a number OF $3N+2$ as a product of $3N+1$, reductio ad absurdum.

IN case 4 I got a prime of $3N+2$ not on the original list, reductio ad absurdum to a .

Hence 1,2,3,4 is false and b is true.

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