Re: Interpolation in Sobolev spaces

Source: http://sci.tech−archive.net/Archive/sci.math/2006−03/msg02381.html

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- *Date*: 14 Mar 2006 05:48:13 −0800

maxsim wrote:

> Hi, I am confused about the pointwise properties of Sobolev functions.
> Does an interpolation constraint like f(x_i)=y_i make sense in Sobolev
> spaces?
> I guess not, but I can't construct two functions in a Sobolev space
> which are not pointwise equivalent but the same w.r.t. the Sobolev
> space i.e. the norm of their difference is zero.
> Does the set on which the Sobolev space is defined play a role?

It depends on the norm whether a point constraint f(x_i) = y_i makes sense in a Sobolev space. Even though we are defining the norm on equivalence classes of functions defined a.e., with sufficiently "high" norms there is a unique continuous function in the equivalence class and thus pointwise equality is well−defined.

A typical application is to finite element approximation with piecewise linear (continuous) functions. Change the value of the function on any set of measure zero, and from a generalized function (distributional) perspective, it hasn't changed. Of course, at points where you did change the value, the result is no longer continuous.

But on H^1[0,1] the linear functional defined by evaluation at a point on the dense subspace of continuous functions is bounded wrt to the Sobolev norm, and so extends in the usual way to a bounded linear functional on the entire space. So in that sense point interpolation makes perfect sense.

Regarding whether the set on which the Sobolev space is defined playing a role, the answer is yes, at least for interpolation on the boundary of the domain. Bear in mind that Sobolev spaces can in general be "anisotropic" with respect to the number of derivatives in different directions. If we limit attention to the case of equal derivatives in all directions and use only L^2 norms on these, one finds that interpolation on the boundary (via trace theorems) is a bit problematic at re−entrant corners (non−convex domains).

As a result attention is often restricted to domains with very smooth boundaries or Cartesian products of such domains.

Did you have an application in mind that requires something else?

regards, chip

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