Re: hahn banach

Source: http://sci.tech−archive.net/Archive/sci.math/2006−03/msg02624.html

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- *Date*: Wed, 15 Mar 2006 08:24:38 −0600

On 15 Mar 2006 04:18:50 −0800, "Fedor" <malabar_carotte@xxxxxxxxx>
wrote:

> hi all,
>
> I'm searching an example of two convex sets A and B

Disjoint? Closed?

> that cannot be
> separated by an hyperplane.

A closed hyperplane (ie one defined by a _continuous_
linear functional)?

And "separated" in what sense?

> I think they exist but I can't find simple
> examples ..

A = {0}, B = {0} gives an example that answers
the question as you stated it.

Two other examples that may or may not be what you
want; unclear because of the missing details in
the question:

(i) In R^2, let A = upper half−plane union positive real
axis, B = lower half−plane union negative real axis.

A and B are disjoint, convex, and if L is any linear
functional on R^2 then L(A) intersects L(B). Of course
A and B are not closed.

(ii) In R^2, A = closed lower half−plane,
B = {(x,y) : y >= exp(x)}. A and B are closed
convex disjoint, and they cannot be separated
in a weaker sense than in (i): There does not
exist a linear functional L and a number s such
that Lp > s for all p in A and Lp < s for all
s in B. On the other hand there _does_ exist
L such that L(A) and L(B) are disjoint (the
point here being you have to specify what
you mean by "separated"...)

thanks,
Fedor.

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David C. Ullrich
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