Re: Logarithm of transfinite numbers

Source: http://sci.tech−archive.net/Archive/sci.math/2006−03/msg02798.html

- *From*: stephen@xxxxxxxxxx
- *Date*: Thu, 16 Mar 2006 03:54:49 +0000 (UTC)

matt271829−news@xxxxxxxxxxx wrote:

> stephen@xxxxxxxxxx wrote:
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> > matt271829−news@xxxxxxxxxxx wrote:
> >
> > > Randy Poe wrote:
> > >
> > > > matt271829−news@xxxxxxxxxxx wrote:
> > > >
> > > > It doesn't seem right because it isn't right.
> > > > The bit string
> > > > representations of natural numbers, whatever
> > > > representation
> > > > you choose, consist of only a countable
> > > > subset of the
> > > > uncountable bit strings of length aleph_0.
> > > > For instance
> > > > you might choose to represent the natural
> > > > numbers as
> > > > ...001, ...010, ...100, ...1000, i.e, infinite bit
> > > > strings where only
> > > > a single bit is nonzero.
> > >
> > > Yes, we *can* represent them this way, but we can already
> > > list them as
> > > 1,2,3,4... which we know is countable. You're just putting
> > > one bit in
> > > the positions 1,2,3,4... so what new does this tell us?
> > >
> > > Imagine instead that we *do* represent them in proper binary
> > > form. We
> > > still need aleph_0 bits, and every arrangement of those
> > > aleph_0 bits

seems to map to a unique natural number.

No, every arrangement of those aleph_0 bits does not seem to map
to a unique natural number. Which natural number is mapped to
..1111111111111
or
..0101010101010

I assume ".." means a (countably) infinite series. They would both have
to be non−finite, I suppose, if that's what you mean.

Yes. It is difficult to actualy write an arrangement of aleph_0
bits in its entirety. :)

Are you saying,
very roughly speaking, that the problem is that the "number infinity"
appears infinitely many times in all those arrangements of aleph_0
bits?

I suppose that is very roughly what I am saying. More precisely,
there are lots of arrangements of aleph_0 bits that do
not correspond to a natural number. There is no "number infinity"
in the natural numbers. Every natural number can be represented
by an arrangment of aleph_0 bits that contains only a finite number
of 1's. Not every arrangement of aleph_0 bits only contains
a finite number of 1's.

Only the bit strings which have an infinite number of leading
zero's correspond to a natural number, and there are only aleph_0
such bit strings.

This is where it goes wrong,
and where the flaw in the logic must lie. It seems to me that
we need
to somehow show that this one−to−one mapping can't be
made.

That is what Cantor did all those years ago.

> Didn't he go bonkers or something? I can't say I'm surprised!

Probably because he had a vision of Usenet and the endless
threads arguing about what is in essence a very simple idea.

Stephen
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