

Re: eigenvalues on the unit circle

Source: <http://sci.tech-archive.net/Archive/sci.math/2006-03/msg03162.html>

- *From:* israel@xxxxxxxxxxx (Robert Israel)
 - *Date:* 17 Mar 2006 22:38:25 GMT
-

In article <1142603926.360117.134770@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx>, pkg <Patric.Gloede@xxxxxxxxxxx> wrote:

I have a function $K(\cdot, \cdot) : S \times S \rightarrow \mathbb{R}$, where S is a countable abelian group. One can see K as an operator acting on $l^2(S)$ and mapping again into $l^2(S)$, where l^2 is the Hilbert space of real square summable sequences, in the following way:

$$K m : l^2 \rightarrow l^2, K m = \sum_{x \in S} K(\cdot, x) m(x)$$

(probabilists may think of the Green function of a random walk!). I know that it is selfadjoint, bounded, and positive, and for $K(\cdot, \cdot)$ seen as a function on $S \times S$ we have

$$\sup_x \sup_y \sum_{z \in S} K(x,z) K(z,y) < \infty.$$

That's implied by the fact K is a bounded operator. If e_x is the standard unit vector ($e_x(z) = 1$ for $z = x$, 0 otherwise), we have $K(x,z) = \langle e_x, K e_z \rangle$ and $\sum_z K(x,z) K(z,y) = \sum_z \langle K^* e_x, e_z \rangle \langle e_z, K e_y \rangle = \langle K^* e_x, K e_y \rangle$ so your double sup is at most $\|K\|^2$.

In addition $K(\cdot, \cdot)$ is symmetric.

Must be, if the operator is self-adjoint.

My question is now: What would be sensible criteria in order to check whether K has eigenvalues on the unit circle (and then because of the other properties $\|K\|$ would be an eigenvalue)?

If K is self-adjoint, the only possible eigenvalues on the unit circle are -1 and 1 .

Re: eigenvalues on the unit circle

Robert Israel israel@xxxxxxxxxxxx

Department of Mathematics <http://www.math.ubc.ca/~israel>

University of British Columbia Vancouver, BC, Canada

.