Re: Ping: Jose Carlos Santos

Source: http://sci.tech−archive.net/Archive/sci.math/2006−03/msg03316.html

- *From*: "Dave L. Renfro" <renfr1dl@xxxxxxxxx>
- *Date*: 18 Mar 2006 11:44:51 −0800

Dave L. Renfro wrote:

> > Godfrey Harold Hardy, A COURSE OF PURE MATHEMATICS,
> > 9'th edition, Cambridge University Press, 1947. [see pp. 52−57]

José Carlos Santos wrote:

> Concerning this one, since I owe a copy of the 10th edition,
> could you please provide the section number(s) instead?

Hummm...I have the 10'th edition also. However, I copied this particular reference from an old post of mine, which I suppose was made before I bought a copy of Hardy's book myself (about 2 years ago) and had to use a library's copy.

See Sections 26−28. In particular, Exercises 14−16 in Section 28 give the same idea that Pierpont used to show that no function with a positive period can be periodic (via a contradiction to the existence of a minimal monic polynomial for the function). Of course, this doesn't give the result for any open interval, but Speck manages to get the stronger open interval version in an elementary and relatively simple way. ("Elementary" refers to the level of mathematics used, and "simple" refers to the complexity of the proofs.)

Dave L. Renfro wrote:

> James Pierpont, THE THEORY OF FUNCTIONS OF REAL VARIABLES,
> Volume 1, Ginn and Company, 1905. [see pp. 123−137]
> http://historical.library.cornell.edu/math/math_P.html

José Carlos Santos wrote:

> This link doesn't seem to be working.

You might have to copy and paste the URL into another internet browser window, although it works for me by just clicking on the link in my post at

http://groups.google.com/group/sci.math/msg/92152c8b1df157cb

If this still doesn't work, go to

http://historical.library.cornell.edu/

Click on the picture above "Historic Math Book Collection", then click "Browse", then (making sure you're in the "Sorted by Author" format) click on "PQ", then look for Pierpont.

By the way, you might also be interested in a more extreme type of transcendental function. A function y of x is said to be "transcendentally transcendental" on an interval (a,b) if P(x, y, y', y", ..., y^(n)) is not identically zero on (a,b) for every positive integer n and every nonzero polynomial P of n+1 variables with rational function coefficients. In other words, y doesn't satisfy any algebraic differential equation (even non−linear). None of the elementary transcendental functions have this property, and most of the higher functions in mathematical physics don't either. However, in 1887 Holder proved that the gamma function is transcendentally transcendental. (I'm not sure, but I think Holder was also the first to formulate this property.) A nice survey paper of this topic is:

Lee Albert Rubel, "A survey of transcendentally transcendental functions", American Mathematically Monthly 96 (1989), 777−788.

Some of the older papers, including Holder's original paper and some by E. H. Moore (1897 −− this is where the term 'TT' originates) and J. F. Ritt (1923, 1926), are in Math. Annalen, and thus are available on the internet. There's also a 1902 paper by Edmond Maillet in Bulletin de la Societe Mathematique de France (Vol. 30, pp. 195−201) that's on the internet.

Mathematische Annalen
http://dz−srv1.sub.uni−goettingen.de/cache/toc/D25917.html

Bulletin de la Société Mathématique de France
http://www.numdam.org/numdam−bin/feuilleter?j=BSMF

http://www.google.com/search?as_epq=transcendentally+transcendental

http://books.google.com/books?as_epq=transcendentally+transcendental

http://scholar.google.com/scholar?as_epq=transcendentally+transcendental

Dave L. Renfro

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