Re: Logarithm of transfinite numbers

Source: http://sci.tech−archive.net/Archive/sci.math/2006−03/msg03452.html

- *From*: matt271829−news@xxxxxxxxxxx
- *Date*: 19 Mar 2006 09:23:57 −0800

imaginatorium@xxxxxxxxxxxxx wrote:

> matt271829−news@xxxxxxxxxxx wrote:
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> > imaginatorium@xxxxxxxxxxxxx wrote:
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> > > matt271829−news@xxxxxxxxxxx wrote:
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> > > > Virgil wrote:
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> > > > > In article <1142680417.743195.20160@xxxxxxxxxxxxxxxxxxxxxxxxxxxx>, matt271829−news@xxxxxxxxxxx wrote:
> > > > >
> > > > > > cbrown@xxxxxxxxxxxxxxxxx wrote:
> > > > > >
> > > > > > > matt271829−news@xxxxxxxxxxx wrote:
> > > > > > >
> > > > > > > > Randy Poe wrote:
> > > > > > >
> > > > > > > <snip>
> > > > > > >
> > > > > > > > > The number of bits can't be finite, since

for any finite value, only finitely many natural numbers can be represented. Yet the smallest infinite cardinal is aleph_0. Therefore there is no representation with less than aleph_0 bits which is large enough for all the natural numbers.

Yes I know, and this is what, to me, "doesn't seem quite right". Aleph_0 bits gives us vastly more strings than we actually need. Does it not, even just a *little* bit, seem "not quite right" to you too? Or do you not see any scope at all for seeing a problem here?

You are mistakenly thinking that the

set of all binary sequences is the same as the set of all binary sequences with finite support.

What do you mean by "with finite support"?

The set of bit positions at which there are non−zero bits is finite.

<snip>

If so, then no, I am not thinking this.

Do tell us what you _are_ thinking <g>!

I'm thinking that we should introduce aleph[−1], aleph[−2], aleph[−3], ... But where that would lead I don't know. Maybe quickly to a contradiction.

What does "introduce" mean?

In this case it really just means speculate that such things can be defined along the lines of aleph[−1] = log(aleph[0]), aleph[−2] = log(aleph[−1]) etc., with a suitable definition of log of course. My technical ability is not up to the job of formulating this with any precision, but the principle is similar, if you like, to x^2 = −1. We suppose that solutions exist, call them +/−i, and see what happens. In that case something very fruitful indeed happens. (I am not claiming that in the aleph case something fruitful *will* happen... it's just an idea.)

> I could define aleph_0 (for example) as the equivalence class under the bijection relation of sets that can be counted against a ditty without ever ending. (You recite the ditty "un−deux−trois" whatever, and are guaranteed to get to every element of the set eventually, but the process never stops.) This isn't a standard formal definition, but I think it is enough to support aleph_0 on the basis of elementary set theory. Where would this "aleph_(−1)" come from? It's a rather Orlovian approach to go round "declaring" this that or the other, just because it looks like helping to produce the local result you want at the moment.

There is nothing wrong with "declaring" something (i.e. saying something is true by definition) and seeing what happens. You can declare anything you like in maths, provided it's all done rigorously and in a non−contradictory manner (which I haven't done, of course). You might end up with something that obviously doesn't hang together, or you might end up in la−la−land, or you might, occasionally, end up with something interesting. Either way you have to start with an open mind. You can't just say "this system that I'm familiar with doesn't have such and such a concept; therefore no system can".

> Of course if you managed to define something in the surreals or whatever that appeared to be log(aleph_0), hmm, you could call it what you like, but if I (semi−)understand correctly, such a thing would be in ordinal arithmetic, not cardinal arithmetic. So it wouldn't really be _reasonable_ to give it an 'aleph' name.

I am sure this idea will have been explored before, but I'm nowhere near knowledgable enough about the subject to know how to map it to something in existing theory. Possibly it is easily shown to lead nowhere and therefore easily discarded.

> After all, aleph_0 is the first infinite cardinal − how can there be another infinite cardinal before the first one?

Perhaps there *isn't* a first one!

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