Re: Probability in an infinite sample space

Source: http://sci.tech−archive.net/Archive/sci.math/2006−03/msg03763.html

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- *From*: "Peter Webb" <webbfamily−diespamdie@xxxxxxxxxxxxxxx>
- *Date*: Tue, 21 Mar 2006 15:47:04 +1100

"Dave Seaman" <dseaman@xxxxxxxxxxxx> wrote in message
news:dvmg28$8la$1@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

> On Tue, 21 Mar 2006 00:14:00 +1100, Peter Webb wrote:
>
>> "Dave Seaman" <dseaman@xxxxxxxxxxxx> wrote in message
>> news:dvl745$l1c$1@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
>>
>>> On 19 Mar 2006 19:04:41 −0800, mikeh106@xxxxxxxxxxx
>>> wrote:
>>>
>>>> If you choose a natural number at random,
>>>> that it will be a multiple of
>>>> 3 is there a 1/3, 1/2, or undefined chance?
>>>
>>> That's a meaningless question, because you have not
>>> identified a
>>> probability distribution.
>>>
>>> It is often assumed that if no distribution is specified, then a
>>> uniform
>>> distribution is intended. That can't be the case here, because
>>> there is
>>> no such thing as a uniform probability distribution on a
>>> countably
>>> infinite sample space.
>>>
>>>> Does it mean something to select "at
>>>> random" from an infinite number of
>>>> objects?
>>>
>>> Certainly, provided a probability distribution is specified.
>>> For
>>> example, there is a uniform distribution on the unit interval

[0,1],
which is an uncountably infinite space. You can also have a probability
distribution on a countably infinite space, but it can't be uniform.
For
example, the probability might be P(n) = 2^(−n) for n = 1, 2, 3, ...

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Dave Seaman
U.S. Court of Appeals to review three issues
concerning case of Mumia Abu−Jamal.
<http://www.mumia2000.org/>

You and another poster strengthened "uniform distribution over N" to "uniform distribution over countably infinite", so I realise that my following remarks are almost certainly wrong.

Consider the set of reals [0,1] that has a terminating decimal representation. Consider the same probability distribution over this as for
all Reals [0,1] uniformly. Isn't this a uniform distribution over a countably infinite set?

It isn't a probability distribution. The total weight is 0, not 1.

Probability distributions are required to be countably additive, but they are
not *uncountably* additive.

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Sorry about the questions, but this is still bugging me.

Lets call the set of Reals of [0,1] as R, and the set of non−terminating decimals within [0,1] as D.

I looked for the definitions of a probability distribution and found this:

http://en.wikipedia.org/wiki/Kolmogorov_axioms

My probability distribution fails condition 3, because the sum of the probabilities of an infinite number number of discrete elements is always zero.

However, this strikes me as a definition which is explicitly designed to make distributions like mine invalid, but allowing similar distributions over uncountable sets.

For example, if we removed the word "countable" from the definition, there would be no uniform distributions over R. Alternatively, if we constrained the intervals to have non−zero probabilities, then my distribution over D would comply.

So I accept that my distribution does not meet the definition, but this definition seems quite arbitrary to me, as its only effect that I can see is to eliminate distributions of the type I propose.

Putting aside this definition, my distribution seems just as well behaved as the standard uniform distribution over R. The probability that an element of D is in [0.1, 0.2] is 0.1, just as it is for the Real case, indeed all probabilities work out to be exactly the same, in both cases the probability of a discrete event is zero, and the probability of a number X being in the range [a,b] is b−a.

Sure, I can't form the sum of the probability of a randomly selected element of D being less than 0.5 by forming the sum of every discrete event where X<0.5 and adding them up, but nor can we for R − its just that the definition (Axiom 3) gives uncountable sets a get−out−of−jail free card, whilst explicitly denying this for countable sets.

My distribution seems to walk like a duck, quack like a duck and swim like a duck.

Why does Axiom 3 single out countable sums but not uncountable sums? Put slightly differently, what would "break" if the definition dropped the requirement that the sum be countable, but instead imposed a requirement that the probabilities being summed over were non−zero? This slightly changed definition appears to work in exactly the same manner, except that it also allows distributions over countable sets (and is hence more powerful).

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