Re: fourier transform in higher dimensions

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- *From*: iredshift@xxxxxxxxx
- *Date*: 2 Apr 2006 16:02:05 −0700

Stephen Montgomery−Smith wrote:

> iredshift@xxxxxxxxx wrote:
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> > Stephen Montgomery−Smith wrote:
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> > > iredshift@xxxxxxxxx wrote:
> > >
> > > > Hi,
> > > >
> > > > I am studying transform methods for solving pdes and I am having
> > > > trouble seeing how transform properties for one variable generalize to
> > > > higher dimensions. For instance, i can show that:
> > > >
> > > > F(u'(x)) = (i*w)*F(u(x))
> > > > F(u''(x)) = (i*w)^2*F(u(x))
> > > >
> > > > or something similar, depending on how you define the fourier
> > > > transform. I can also see that in multiple dimensions:
> > > >
> > > > F(laplacian(u(x1,...,xn))) = (I*w)^2*F(u(x1,..,xn))
> > > >
> > > > since the laplacian just gives a scalar for u: R^n−>R and the transform
> > > > is linear
> > > >
> > > > But what about vector functions? For instance,
> > > > is F(Grad(u(x1,..,xn))) = (i*w)*F(u(x1,..,xn)) ?

If so, how can I see that this indeed should
be the case.

Thank you for your help!

The Fourier transform of u(x1,...,xn) is a function of
w1,...,wn. Thus
F(Grad u) = I (w1,...,wn) F(u)
and
F(Lap u) = ((Iw1)^2+...(Iwn)^2) F(u).

Stephen, thanks for your help! I am just slightly confused by the
notation. Let's say I have a function u: R^n−> R of period 1 and the

You mean "period 1 in all n co−ordinates" because different coordinates
might have different periods.

vectors:

X = (x1,..,xn)
K = (k1,...,kn)

Now,

F[u(X)](K) = integral{ exp(−2*pi*i*X*K) * f(X) dX }

So in this notation I write,

Lap u(X) = (d^2/dx1^2)u + ... + (d^2/dxn^2)u which is a scalar and

F[Lap u(X)] = (−2*pi*i*K)*(−2*pi*i*K) * integral{ exp(−2*pi*i*X*k) *
f(X) dX } =

= −4*pi^2*|K|^2 * F[u(X)](K)

So how would I express,

F[Grad u(X)] = F[((d/dx1)u,...,(d/dxn)u)]?

= −2*pi*i (k1,...,kn) F[u].

Ok thanks, yes I see, if it's not period 1 in all n then the −2*pi*i can't factor. Although I guess by a u−substitution you can bring the factors outside the integral. (i.e. multiply by something like 1/(p1*p2*...*pn))

So I guess it's:

F[Grad u(X)](k1,...,kn) = F[((d/dx1)u,...,(d/dxn)u)](k1,...,kn) = −2*pi*i * F[u](k1,...,kn)

i.e. where the last part if the fourier transform of u evaluated at the vector (k1,..,kn).

But say I have some vector V = (v1,...,vn) and I do:

F[V*Grad u(X)] = v1*F[(d/dx1)u](K) + ... + v2*F[(d/dxn)u](K) by linearity

So now does that = (−2*pi*i)*F[u](K)*(v1 + ... + vn)?

I guess my question comes down to whether in the n−dim case,

F[(d/dxA)u](K) = F[(d/dxB)u](K) = (−2*pi*i)*F[u](K) for 1<A<B<n assuming the periods in the variables are 1.

Thanks!

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