Re: Compute eigen values/vectors of a 22k x 22k matrix

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- *Date*: Mon, 03 Apr 2006 09:27:28 +0300

On Mon, 03 Apr 2006 00:16:57 −0400, quasi <quasi@xxxxxxxx> wrote:

> On Sun, 02 Apr 2006 22:34:23 −0400, quasi <quasi@xxxxxxxx> wrote:
>
>> On 2 Apr 2006 19:28:31 −0700, "rveloso" <rvelosoo@xxxxxxxxx> wrote:
>>
>>> Hi all, wonder if anyone has suggestion about how to
>>> compute eigen
>>> values/vectors of a 22k x 22k matrix. The matrix only has 0's
>>> and 1's
>>> and is symmetric.
>>
>> Any CAS program (for example Maple, Mathematica, or Maxima) should be able to do it easily.
>>
>> While I prefer Maple or Mathematica, they both cost money whereas Maxima is free.
>
> Whoops −− now that I see R.G. Vickson's reply, I realize that I missed the "k". I read it as 22 x 22 which is not big at all.
>
> If k means 1000, then a 22k x 22k matrix is a big matrix and might be too big for the CAS progs. Maybe, maybe not. They can at least store such a matrix if you have enough RAM and/or disk space, but the problem will then be time. CAS packages are slow relative to custom software written in compiled languages such as C or C++.
>
> The space complexity of your problem is O(n^2), but the time complexity is O(n^3) (actually a little less −− O(n^2.81), I think, with specialized algorithms).

Strassen's algorithm has nothing to do with finding eigenvalues. You never ever do any matrix−matrix multiplications or decompositions to matrices of that size.

> O(n^3) for your problem means some constant times 10^13. Given the simple requirements of the problem, the constant is probability not too big, but with that time complexity, I wouldn't even bother with a CAS.
>
> Instead, I would write a custom program in C. You will also need a bignum package or custom bignum routines since even though your matrix entries are all 0,1, the characteristic polynomial is almost certain to have coefficients outside the range of standard integer sizes. Also, even if the eigenvalues get approximated as decimals (or complex numbers with decimal components), you definitely want to get the coefficients of the characteristic polynomial exactly as integers. All approximations should be deferred to the root finding phase.

OK, that is not going to get you anything but garbage. You don't find the eigenvalues of any matrix by computing anything with the characteristic polynomial.

The general methods for finding eigenvalues of large matrices are iterative. Hermitian Lanczos or something might work in this case if the matrix is either sparse enough to keep in memory or, failing that, the matrix−vector product with it can be effectuated efficiently.

The iteration will reduce the system to a smaller tridiagonal form which has (approximately) some subset of the largest eigenvalues of the actual matrix as its eigenvalues. If you want all of them, you have to iterate all the way to the end and then find the eigenvalues of that tridiagonal matrix. It also gives you a method for finding the eigenvectors.

> My gut feel is that finding the eigenvalues of a 22k x 22k matrix is within reach of modern computers, but it might take days, weeks, or months, depending on the computing power (ram, cpu speed, etc.) and the software used. However, it might be possible to get a substantial speedup by using multiple computers together with algorithms oriented to parallel processors.

Depends a lot on the sparsity and structure of the matrix.

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