Re: Calculus XOR Probability

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- *From*: Tony Orlow <aeo6@xxxxxxxxxxx>
- *Date*: Tue, 11 Apr 2006 11:55:48 −0400

cbrown@xxxxxxxxxxxxxxxxx said:

> Tony Orlow wrote:
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>> cbrown@xxxxxxxxxxxxxxxxx said:
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>>> Tony Orlow wrote:
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>>>> cbrown@xxxxxxxxxxxxxxxxx said:
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>>>>> Han de Bruijn wrote:
>>>>>
>>>>>> cbrown@xxxxxxxxxxxxxxxxx wrote:
>>>>>>
>>>>>>> What does surprise me is that you don't see the obvious parallel between my reasoning that since lim (n*(2/n))=2, therefore the diagonal has

length 2; and your reasoning that since the lim (n*(1/n))=1, that therefore there must exist a uniform distribution on the naturals.

<snip>

If you use 2 steps, you still get 2.

If you use a 2 element set, you still get a total probability of 1.

Uh, yeah, except that 1 is the value you want for the probability of the entire sample. 2 is not the value you want for the diagonal.

Han seems unbothered by the fact that 1 is not the value you get from the sum of a countable number of 0's; he simply "wants" it to be 1. And lo and behold, that is what his argument affirms, just as mine affirms that the length of the diagonal is exactly what I "want" it to be: 2.

No, he doesn't just "want" it to be 1. That is the probability of something which is definitely true. If one of the elements is to be chosen from the set, then the probability of one being chosen is 1. If there are multiple choices, then that 1 representing the fact that one of those will be chosen is defined to be the sum of the probabilities of each. That's how probability works, and the question is why the standard system can't accomodate the infinite case, with an infinite number of equally likely outcomes, one of which will definitely happen. The probability of each is 0 in standard analysis, and yet each has a chance of being chosen, and so the probability is really nonzero. It's only zero in standard finite analysis, but where you are talking about an infinite set of possibilities, you are outside of standard territory, and the answer is nonstandard: each possiblity has an infinitesimal probability, and that infinity of infinitesimal values sums to 1 as expected. So, n*1/n=1 holds in the infinite case, and there is no reason to expect it to fail. Is there?

> > > > As you use more and more
> > > > steps, it looks more and more like a diagonal
> > > > line, but the length stays the
> > > > same as when it did not.

> > > As you use more and more elements, it "looks" more and
> > > more like a
> > > uniform distribution on the naturals, but the total probability
> > > stays
> > > the same as when it did not (sic; i.e., whatever that means).

> > The sum of all outcomes should be 1 if they are mutually exclusive, equally probable, and one must be chosen. So, it's GOOD that n*1/n equals 1 for all (finite) n.

> Similarly, if the sum of all steps "should be" 2, that's what I get as I travel along the tiny stair steps. So, it's GOOD that n*2/n equals 2 for all (finite) n; in fact, I find it extremely good evidence that in the limit, it will be 2 as well.

If you construct a stairway there, well, it will be 2. That's not a diagonal line. So, fine, your derivation is correct. No matter what size stairs you use, or how many, it will always add up to 2. Nice proof. It just has nothing to do with a diagonal line. The sum of the probabilities of all mutually exclusive possibilities summing to 1, on the other hand, is a cornerstone of probability theory, and something that should be considered always true.

> > > > You are not approximating the length of the
> > > > diagonal. That sum is never
> > > > equal nor getting closer to the answer.
>
> > > You are not approximating a uniform distribution on the
> > > naturals. That
> > > sum (1) is never equal to or getting closer to "the answer"
> > > (which is
> > > either 0, since the sum of a countable number of 0's is 0; or
> > > infinite
> > > by the archimedean property of the reals).
>
> > Uh, no. If one of the outcomes is to be selected, then the probability of all
> > outcomes should sum to 1. The fact that you get a sum of 0 in the infinite
> > case
> > indicates an error. The sum is correctly 1 for all finite n. The sum is
> > correctly 1 for infinite n as well, but the the probability of each outcome is
> > 1/n, which in the infinite case is infinitesimal, not absolute zero.
>
> By exactly the same logic:
>
> If I travel along the stair steps, no matter how small, then sum of all
> the stair steps "should" sum to 2. The fact you seem to think that the
> distance in the infinite case is actually sqrt(2) indicates an error.
> The sum is correctly 2 for all finite n. The sum is correctly 2 for
> infinite n as well, but the length of each stair step is 2/n, which in
> the infinite case is an infinitesimal hodon [*], not absolute 0.

Yeah, sure. Like I said, it's not the diagonal line, even though it starts to
look like it. You're talking about a fractal dimension on the line, basically.
What does that have to do with probability? It's just an attempt to change the
subject into something ridiculous and trying to say it's the same argument.

> Is the parallel making sense to you yet? Do you see /any/ difference in
> these /arguments/, besides the fact that one gives an answer you "want"
> or "should get", whereas the other doesn't?

Yes, I do. The difference is that you're trying to say your example is about a
diagonal line, and it's not, whereas Han's trying to say his example is about
probability in an infinite set, which it is, to whatever extent he actually
means "infinite". ;−)

[*] e.g., http://plato.stanford.edu/entries/geometry−finitism/

> So, the question here
> is, why would you think it gives a correct
> answer at oo, if it gives an equally
> incorrect answer for all finite nnumber of
> steps? The limit of the error is not
> 0.

So, the question here is, why would you think it gives a correct answer
at oo, if it gives an equally incorrect answer for all finite number of
steps? The limit of the error is not 0.

Uh, excuse me. Maybe I'm getting confused, but maybe not. If there is a set of
n equally likely outcomes, and one is picked, is not the chance of one being picked equal to 1, and is this not the sum of n individual probabilities of 1/n? Is the chance of picking any given natural from 1 to a million 1 millionth? What error is there in "n*1/n=1" for any finite n? None, so what is the parrot act all about? Is this what you call analysis? Please try to answer the question, sincerely.

Alright, I'll stop − but I encourage you to do it for yourself: Just substitute in your argument "trillions of tiny steps" and "What error is there in "n*2/n" for any finite n?"

There's no error, if you're talking about stairs. If you're taslking about a diagonal, you have to take a different approach. I don't see much of a parallel outside of superficial argument structure. There's more than sematics to this question.

Note that:

* No one is claiming anything other than that, in a finite set of n equal outcomes, each outcome has probability 1/n; just as no one would claim anything other than that if there are (finite) n steps, each

> tread and riser has length 2/n.

Uh huh, and if you have an infinite number of infinitesimal steps, you're going to have a sum of 2 too.

> * No one is claiming that for finite n, n*1/n equals something other than 1, anymore than than any one would claim that n*2/n equals something other than 2.

Good, so no one's totally insane....

> So there's no reason to repeat these assertions; they are accepted.
>
> Each of the claims you have made so far for the correctness of Han's argument can be mirrored as an equally valid claim for correctness of my argument; just substitute "2" for "1" and "sqrt(2)" for "0", and "finite number of stair steps" for "finite set of outcomes".

Great, so it's true for the infinite staircase. And?

> But my /conclusion/ is so obviously false (by appeal to Pythagoras), there must be something wrong with my /argument/.

Yes, it has nothing to do with the diagonal line. Your line is still vertical and horizontal.

> I claim that if you can figure out why the /argument/ is wrong (not just the conclusion), then you will also see why Han's argument is wrong (above and beyond the fact that his conclusion is also independently false, although perhaps not as obviously, by appeal to Archimedes).

But it's not. Archimedean principle is mitigated at the infinitesimal level by the fact that neighboring infinitesimals do not constitute distinct standard reals, and so on the finite level, any discreteness of the infinitesimals, if they are taken to be discrete, does not violate Archimedean principle on that level. Further, on the infintiesimal level, Archimedean principle CAN be preserved by defining midpoints in the infinitesimal intervals, because at that level, the endpoint ARE distinct values in their nonstandard sense.

Why is your conclusion wrong? It's not, if you're talking about a line broken into vertical and horizontal elements. It will always be equal to the sum of the vertical and horizontal distances traveled, because it's NOT the diagonal between the starting and ending points.

> > > If you have n possibilities all mutually exclusive and equally likely, and one of them must occur, then the chances that any given one will occur is 1/n, so that the probabilities of all will sum to 1, as expected.

> > If you have n steps in the diagonal, the length of each tread/riser is
> > 2/n, so that the total length will sum to 2, as expected.

> So, you EXPECT the diagonal to be equal to 2? Whatever. You're not discussing
> things constructively at all.

This is much closer to the problem:

Why would you think that the answer is anything BUT 2? In fact, the proof shows it /must/ be 2!

Equally, why would the sum of a countable number of equal reals not possibly be 1? In fact, the proof shows it /must/ be 1!

I'm not especially comfortable with the use of the word "countable" in this context. We are not talking about a uniform distribution over the standard naturals. We've already agreed that doesn't exist, since it depends directly on the contradictory "largest finite". What Han is suggesting is that the sum of the probabilities of mutually exclusive events, one of which must happen, is equal to 1, and that this rule should be preserved in all cases, including the infinite case. He is pointing out that standard analysis fails in this regard, and seems amenable to the notion that infinitesimal probabilties solve the problem. The question is, why aren't you?

As n increases
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As n increases without bund (sic), this relationship is preserved and
functions without issue, given expected results.

So, you DO think the diagonal of a unit square is 2. I see. And you think Han and I are cranks.....

No, I don't think that the diagonal is "really" 2. But the logic is /exactly/ the same.

Yeah, and it proves what it sets out to, that a staircase requires risers and treads that sum to the sum of the height and width traversed. There isn't any diagonal line there. In the limit, it appears as one, but it's a fractal line, with a dimension greater than 1.

Han uses this logic to conclude that there is a uniform distribution on the naturals. Someone might claim that Han's conclusion is obviously wrong, because the Archimedean Principle /clearly/ implies that the sum of a countable number of equal real numbers must either be 0 or infinite. Han replies that the Archimedean Principle therefore indicates a flaw in the real numbers.

I don't think Han still thinks there's a uniform distribution over the standard naturals, if he did. You seem to want to talk about "countable" numbers, which draws the naturals into the conversation. But that's a diversion. I don't really see the Archimedean principle having the effect you say it does, but I can see that without infinitesimals, the sum of an infinite number of equal values must be either 0 or infinite. But, that's only without infinitesimals.

I uses the same logic to conclude that the diagonal of a unit square has length 2. Someone might respond that my conclusion is obviously wrong, because the Pythagorean Theorem /clearly/ implies that the

> diagonal is sqrt(2). I reply that the Pythagorean Theorem therefore indicates a flaw in Euclidean geometry.

Yeah, except han started with a tenet of probability theory, and we have a clear idea of how to preserve it, and you started with something that is obviously larger than the diagonal, and a process that does not change that measure in any number of iterations, indicating that you aren't approximating anything at all, but simply proving a constant value for a staircase of any scale.

> But there' no need to go off on a tangent into some kind of bizzare alternative mathematics here. Our grandiose counterclaims are both just /hot air/. Neither reply is valid; because in both cases, the conclusions /don't follow/ from the arguments to start with.

The only conclusion that doesn't follow is the one that your argument had anything to do with measuring the diagonal to begin with. Nice trick that. Not math, but tricky still.

> > > > So, the question here is, why would you
> > > > expect this
> > > > relationship between n 1/n's summing to 1 to
> > > > break down at n=oo?

> > > So the question here is, why would you expect this
> > > relationship between
> > > n 2/n's summing to 2 to break down at n=oo?

> > I wouldn't, but if you're using some technique to approaximate the diagonal, I would expect it to get closer with successive iterations, not maintain the same wrong value.

> What "wrong value"?

The 2 which is obviously larger than the diagonal.

> My argument claims that the right value is 2, and
> that sqrt(2) is wrong (and therefore Euclidean geometry is flawed). Han
> claims the right value is 1, and that 0 is wrong (and therefore, the
> standard reals are flawed). We both maintain our respective right
> values, all the way out to the limit; keeping continuity between the
> finite and the "potential infinite".

Oh, that's a belch of hot air, Chas. How can you deny that the laws of probability define 1 as the probability of something that is definitely true? If we are choosing 1 of a set of elements, the probability that one from that set will be chosen is 1. Do you disagree yet?

Can you deny that the laws of probability say that the probability of one of a set of mutually exclusive possibilities happening is equal to the sum of each of them happening?

Do you disagree that if there is a uniform probability distribution, that each possiblity has an equal probability?

Do you agree that there may be an infinite number of possiblities in some cases?

Does it make sense to you that in an infinite set of possibilities, at least SOME of them must have a probability of 0, even if they have a chance of occurring? Does this 0 mean "no chance at all", or is it simply 0 for lack of a standard treatment of the infinitesimals?

> But you're getting warmer. Consider your use of "technique to
> approximate". Where in my argument do I actually /justify/ my
> assertion that the stairstep really is a valid "technique of
> approximating" the length of the diagonal?

Nowhere.

> Now apply this same question to Han's argument regarding a
> distribution:
>
> Where is it actually /justified/ that "approximating" a uniform
> distribution on the naturals can be accomplished by looking at uniform
> distributions on finite sets of naturals?

Forget the naturals. Assume some infinite n, as Han did originally, and stop changing the subject. Stop making everything a "largest finite" argument. That's the major problem with transfinite set theory. You folks huddle around your golden Aleph_0 statue drawing conclusions from the Void. Pick points and measure the Universe.

> What does it mean, mathematically, to "get closer with successive iterations" to a uniform distribution on the naturals? Exactly how "far away" is a uniform distribution on {1..10} from a uniform distribution on the naturals?

ZZZzzzzzz.........

> This requires /at least/ as much solid justification as claiming that the stair−step approach "approximates" the diagonal in the limit.

Yeah, except it wasn't in Han's original argument. You can put aleph_0 back in its cage now.

> And such justifications must be done carefully. For example, I claim that the stairstep approach approximates the diagonal because the error here can be defined as the total area of difference between the stairstep curve and the diagonal. This area certainly approaches 0 as n approaches infinity. So the stairstep curve, in the limit, will have a 0 area difference with the diagonal, and therefore is the same curve; thus they must have the same length, which by Han's logic must be 2; since "Nature abhors skipping about like a little girl": Glibnits. QED.
>
> Consider: isn't this "exactly how" how we are taught that the length of a continuous curve can be approximated by "infinitesimally small" line segments?

That doesn't work if you use infinitely small line segements that aren't PARALLEL with the curve you are approximating, now, does it? Your example is diagonal−free. Move on. Consider the fact that n*1/n=1 for specific infinite n, and its specific infinitesimal multiplicative inverse.

>> I don't think you even thought for a second before writing. Your loss.

> I didn't realize thinking was required in this newsgroup.

That's becoming kinda obvious, Chas. Some of us are actually trying to answer new questions. ;−)

Cheers − Chas

−−
Smiles,

Tony
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