Re: Calculus XOR Probability

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- *From*: Tony Orlow <aeo6@xxxxxxxxxxx>
- *Date*: Tue, 11 Apr 2006 15:41:56 −0400

Matt Gutting said:

> Tony Orlow wrote:
>
>> cbrown@xxxxxxxxxxxxxxxxx said:
>
> <snip>
>
>>> Han seems unbothered by the fact that 1 is not the value you
>>> get from
>>> the sum of a countable number of 0's; he simply "wants" it to
>>> be 1. And
>>> lo and behold, that is what his argument affirms, just as mine
>>> affirms
>>> that the length of the diagonal is exactly what I "want" it to
>>> be: 2.
>
> Tony:
>
>> No, he doesn't just "want" it to be 1. That is the probability of something which is definitely true. If one of the elements is to be chosen from the set, then the probability of one being chosen is 1. If there are multiple choices, then that 1 representing the fact that one of those will be chosen is defined to be the sum of the probabilities of each. That's how probability works,
>
> Only if you can define a probability distribution, which is the assumption in question here.

Not true. If you can establish a uniform probability distribution, then you can say each possibility has the same probability. If you can formulate the distribution at all, it will allow you to calculate which fraction of the overall probability is assigned to each possiblity. But whether you can determine what the distribution is or not, the overall probability that one will be chosen, given the fact that you're choosing one, is 1, and the probabilities of each mutually exclusive alternative contributes to that sum equal to 1. That's a very basic concept that must hold, even if we don't know what probability distribution we're looking at. If the probabiliies of all the mutually exclusive alternatives in a set sums to zero, then what that says is that no element from the set is selected.

> Tony:
>
> > and
> > the question is why the standard system can't accomodate the infinite case, with an infinite number of equally likely outcomes, one of which will definitely happen. The probability of each is 0 in standard analysis,
>
> Not precisely. It's probably more correct to say that there's no number which can be assigned to that probability. Should there be? I don't see a problem with answering "I can't tell you" to a question of probabilities.

Yes, precisely, the probability assigned to one of a set of equally likely mutually exclusive alternatives is 0% in standard analysis, which is what this is all about. That's not absolute zero, but some infinitesimal value which is not handled by the standard system. So, you don't see a problem with not being able to say, but is that what you prefer?

> Tony:
>
> > and yet
> > each has a chance of being chosen, and so the probability is really nonzero. It's only zero in standard finite analysis, but where you are talking about an infinite set of possibilities, you are outside of standard territory, and the answer is nonstandard: each possiblity has an infinitesimal probability, and that infinity of infinitesimal values sums to 1 as expected. So, n*1/n=1 holds in the infinite case, and there is no reason to expect it to fail. Is there?
>
> Problem here is that you haven't really defined "infinitesimal" in any rigorous and non−circular way.

Immaterial. If n is infinite, then 1/n is infinitesimal. It's smaller than any finite value and yet nonzero, just like the probability we're discussing.

Charles said:

> Similarly, if the sum of all steps "should be" 2, that's what I get as
> I travel along the tiny stair steps. So, it's GOOD that n*2/n equals 2
> for all (finite) n; in fact, I find it extremely good evidence that in
> the limit, it will be 2 as well.

Tony:

> If you construct a stairway there, well, it will be 2. That's not a diagonal line. So, fine, your derivation is correct. No matter what size stairs you use, or how many, it will always add up to 2. Nice proof. It just has nothing to do with a diagonal line. The sum of the probabilities of all mutually exclusive possibilities summing to 1, on the other hand, is a cornerstone of probability theory, and something that should be considered always true.

If one can find a probability distribution for the elements, yes. Otherwise all bets are off (pun intended).

Whether one can determine a distribution or not, there is never 110% chance of something happening. If you have a set of mutually exclusive possiblities, and one of them is going to happen, the probabilities of each happening must sum to 1. If it sums to less than 1, then you have left out some possibliites, and if it sums to greater than 1, then some possiblities are not mutually exclusive.

Charles:

> If I travel along the stair steps, no matter how small, then sum of all
> the stair steps "should" sum to 2. The fact you seem to think that the
> distance in the infinite case is actually sqrt(2) indicates an error.
> The sum is correctly 2 for all finite n. The sum is correctly 2 for
> infinite n as well, but the length of each stair step is 2/n,

which in
the infinite case is an infinitesimal hodon [*], not absolute 0.

Yeah, sure. Like I said, it's not the diagonal line, even though it starts to look like it. You're talking about a fractal dimension on the line, basically. What does that have to do with probability? It's just an attempt to change the subject into something ridiculous and trying to say it's the same argument.

Is the parallel making sense to you yet? Do you see /any/
difference in
these /arguments/, besides the fact that one gives an answer
you "want"
or "should get", whereas the other doesn't?

Tony:

Yes, I do. The difference is that you're trying to say your example is about a diagonal line, and it's not, whereas Han's trying to say his example is about probability in an infinite set, which it is, to whatever extent he actually means "infinite". ;−)

Both Han's proof and Charles' depend on reasoning from the behavior of an increasing, but always finite, number of elements to an infinite number of elements. To that extent, the proofs expose the same types of reasoning flaws.

There's really no flaw in proving an equality between expressions in the infinite case. Inequalities can be invalid in the infinite case, where the difference causing the inequality has a limit of 0 as n−>oo. Here, the flaw was simply that the staircase never was a diagonal line, even if it appears so visually as n−>oo. Chas mentioned approximating lengths of curves with infinitesimals, but those are always parallel to the curve, while his are always at a 45 degree angle, causing the sqrt(2) error by the sine and cosine of that angle.

Charles:

* No one is claiming that for finite n, n*1/n equals something
other
than 1, anymore than than any one would claim that n*2/n
equals

something other than 2.

Tony:

Good, so no one's totally insane....

Charles:

So there's no reason to repeat these assertions; they are
accepted.

Each of the claims you have made so far for the correctness
of Han's
argument can be mirrored as an equally valid claim for
correctness of
my argument; just substitute "2" for "1" and "sqrt(2)" for "0",
and
"finite number of stair steps" for "finite set of outcomes".

Tony:

Great, so it's true for the infinite staircase. And?

Charles:

But my /conclusion/ is so obviously false (by appeal to
Pythagoras),
there must be something wrong with my /argument/.

Tony:

Yes, it has nothing to do with the diagonal line. Your line is still vertical and horizontal.

And Han's increasingly large finite sets are still finite, whereas his conclusion is drawn about something infinite. What's the difference between using finite sums of lengths to draw conclusions about an "infinite" (in

some sense) sum of lengths, and using finite sets of equiprobable elements
to draw conclusions about an infinite set of elements?

No difference whatsoever, and Chas' conclusion that the infinitely fine staircase still requires a line of length 2 is correct, but it's not a diagonal line. The only mistake is thinking it's a diagonal line. the length is 2, and the sum of the probabilities is 1, by proper inductive proof of equality holding for the infinite case. Bith proofs are valid in my opinion.

I claim that if you can figure out why the /argument/ is
wrong (not
just the conclusion), then you will also see why Han's
argument is
wrong (above and beyond the fact that his conclusion is also
independently false, although perhaps not as obviously, by
appeal to
Archimedes).

But it's not. Archimedean principle is mitigated at the infinitesimal level by the fact that neighboring infinitesimals do not constitute distinct standard reals, and so on the finite level, any discreteness of the infinitesimals, if they are taken to be discrete, does not violate Archimedean principle on that level. Further, on the infintiesimal level, Archimedean principle CAN be preserved by defining midpoints in the infinitesimal intervals, because at that level, the endpoint ARE distinct values in their nonstandard sense.

Again, you haven't defined infinitesimals rigorously and non−circularly,
so referring to them in an argument can't lead to valid conclusions.

Waaahhh...

Smaller than any finite but non zero. The multiplicative inverse of an infinity.

Why is your conclusion wrong? It's not, if you're talking about a line broken into vertical and horizontal elements. It will always be equal to the sum of the vertical and horizontal distances traveled, because it's NOT the diagonal between the starting and ending points.

> What gives you the idea that one can "sum" the probabilities of an infinite number of points to get the whole (1) any more than one can add up infinite numbers of vertical and horizontal lengths to get a diagonal length?

Because they add linearly, without the need to be parallel to what they "approximate", since the values are scalar and cannot be anything BUT parallel, as opposed to the vectors that the treads and risers represent. If all your infinitesimal segments were PARALLEL to the diagonal, then indeed they would sum to, guess what, sqrt(2)!

> > > If you have n possibilities all mutually exclusive and equally likely, and one of them must occur, then the chances that any given one will occur is 1/n, so that the probabilities of all will sum to 1, as expected.
>
> > If you have n steps in the diagonal, the length of each tread/riser is 2/n, so that the total length will sum to 2, as expected.
>
> So, you EXPECT the diagonal to be equal to 2? Whatever. You're not discussing things constructively at all.

This is much closer to the problem:

Why would you think that the answer is anything BUT 2? In
fact, the
proof shows it /must/ be 2!

Equally, why would the sum of a countable number of equal
reals not
possibly be 1? In fact, the proof shows it /must/ be 1!

I'm not especially comfortable with the use of the word "countable" in this context. We are not talking about a uniform distribution over the standard naturals. We've already agreed that doesn't exist, since it depends directly on the contradictory "largest finite". What Han is suggesting is that the sum of the probabilities of mutually exclusive events, one of which must happen, is equal to 1, and that this rule should be preserved in all cases, including the infinite case. He is pointing out that standard analysis fails in this regard, and seems amenable to the notion that infinitesimal probabilties solve the problem. The question is, why aren't you?

See earlier comments on "infinitesimal". I'm still awaiting the axiomatization of "infinite" and "infinitesimal" which you said in another thread you were working on. Until I can see that, the answer to your question must be "Because I don't know that it makes sense."

How on Earth can it be that n*1/n<>1? How can that make sense?

<snip>

My argument claims that the right value is 2, and
that sqrt(2) is wrong (and therefore Euclidean geometry is
flawed). Han
claims the right value is 1, and that 0 is wrong (and therefore,
the
standard reals are flawed). We both maintain our respective
right
values, all the way out to the limit; keeping continuity
between the
finite and the "potential infinite".

Oh, that's a belch of hot air, Chas. How can you deny that the laws of probability define 1 as the probability of something that is definitely true? If we are choosing 1 of a set of elements, the probability that one from that

> > set will be chosen is 1. Do you disagree yet?
>
> I don't know about him, but I don't. Yet.

> > Can you deny that the laws of probability say that the probability of one of a set of mutually exclusive possibilities happening is equal to the sum of each of them happening?
>
> As phrased, yes, I can and do deny it.

What in the world could you be objecting to? Say we have a set of 3 mutually exclusive possibilities, A, B and C. Is the probability P(A or B or C) not equal to P(A)+P(B)+P(C)? At least in the finite case, you must agree.

> > Do you disagree that if there is a uniform probability distribution, that each possiblity has an equal probability?
>
> No, I don't disagree. But I thought that we were agreed there was no probability distribution?

Forget the naturals already. That wasn't Han's original idea. Set theorists brought that in to confound the topic. Drop it.

> > Do you agree that there may be an infinite number of possiblities in some cases?
>
> Yes.

> > Does it make sense to you that in an infinite set of possibilities, at least SOME of them must have a probability of 0, even if they have a chance of occurring? Does this 0 mean "no chance at all", or is it simply 0 for lack of a standard treatment of the infinitesimals?

It's not 0, as I understand it. It's simply that there's no well−defined way
to talk about a number having "a chance of occurring". I don't have a problem
with that. Should I?

You can accept that as alright for your purposes, but that doesn't mean it's wrong to consider. If you don't care about how to talk about it, then don't. But don't sit around telling people they are being dumb for discussing it. Infinitesimal probabilities are the way to discuss it. If you don't like that, don't discuss it. Personally, I think it's crucial for the next step in the study of infinity and the development of the foundations of math. So, if you don't mind, I think I shall engage in such conversation to see what actual problems it causes. So far, I don't see any.

<snip>

Consider: isn't this "exactly how" how we are taught that the
length of
a continuous curve can be approximated by "infinitesimally
small" line
segments?

That doesn't work if you use infinitely small line segements that aren't
PARALLEL with the curve you are approximating, now, does it? Your
example is
diagonal−free. Move on. Consider the fact that n*1/n=1 for specific infinite
n,
and its specific infinitesimal multiplicative inverse.

See above comments on infinitesimals.

Yeah, I saw.

"I don't know what you mean by 1/n being infinitesimal for infinite n because you haven't rigorously developed a theory I can microscopically analyze, and I can't understand simple concepts like lim(x−>oo: 1/x)−>0."

Matt

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Smiles,

Tony
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