Re: Calculus XOR Probability

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* *From*: Tony Orlow <aeo6@xxxxxxxxxxx>
* *Date*: Wed, 12 Apr 2006 13:39:16 −0400

Matt Gutting said:

> Tony Orlow wrote:
>
>> Matt Gutting said:
>>
>>> Tony Orlow wrote:
>>>
>>>> cbrown@xxxxxxxxxxxxxxxxx said:
>>>
>>> <snip>
>>>
>>>>> Han seems unbothered by
>>>>> the fact that 1 is not the
>>>>> value you get from
>>>>> the sum of a countable
>>>>> number of 0's; he simply
>>>>> "wants" it to be 1. And
>>>>> lo and behold, that is what
>>>>> his argument affirms, just as
>>>>> mine affirms
>>>>> that the length of the
>>>>> diagonal is exactly what I
>>>>> "want" it to be: 2.
>>>
>>> Tony:
>>>
>>>> No, he doesn't just "want" it to be 1. That is
>>>> the probability of something
>>>> which is definitely true. If one of the
>>>> elements is to be chosen from the set,
>>>> then the probability of one being chosen is 1.
>>>> If there are multiple choices,
>>>> then that 1 representing the fact that one of
>>>> those will be chosen is defined
>>>> to be the sum of the probabilities of each.
>>>> That's how probability works,

> > > Only if you can define a probability distribution, which is the
> > > assumption in
> > > question here.
>
> > Not true. If you can establish a uniform probability distribution, then you can say each possibility has the same probability. If you can formulate the distribution at all, it will allow you to calculate which fraction of the overall probability is assigned to each possiblity. But whether you can determine what the distribution is or not, the overall probability that one will be chosen, given the fact that you're choosing one, is 1, and the probabilities of each mutually exclusive alternative contributes to that sum equal to 1. That's a very basic concept that must hold, even if we don't know what probability distribution we're looking at.
>
> It must hold, even if we don't know what probability distribution we're talking about, *assuming* that we're looking at a probability distribution. As I said, the question is whether one can be defined here. You appear to be assuming it can. I'm not.

Somethign tells me you're still trying to deal with a uniform probability distribution over tha naturals. That's impossible, but given any set of n equally likely outcomes, if exactly one of them will occur, each has a 1/n chance of being the one to occur. To assume you have a set of possibilities WITHOUT a probability distribution is just to say you have no idea what any of the individual probabilities is. That has no effect on the fact that if exactly one is to occur, the probabilities of each sum to that 1.

> > > Tony:
> > >
> > > > and
> > > > the question is why the standard system can't
> > > > accomodate the infinite case,
> > > > with an infinite number of equally likely
> > > > outcomes, one of which will
> > > > definitely happen. The probability of each is
> > > > 0 in standard analysis,
> > >
> > > Not precisely. It's probably more correct to say that there's no
> > > number which
> > > can be assigned to that probability. Should there be? I don't
> > > see a problem with
> > > answering "I can't tell you" to a question of probabilities.

Yes, precisely, the probability assigned to one of a set of equally likely mutually exclusive alternatives is 0% in standard analysis, which is what this is all about. That's not absolute zero, but some infinitesimal value which is not handled by the standard system. So, you don't see a problem with not being
able to say, but is that what you prefer?

The difficulty here is that you haven't really shown an explicit and consistent way of defining and working with infinitesimals. That must come first, before you begin using them to explain things.

What do you want, a number system? By all means, use the T−riffics. If we have 1:000...000 equally likely possibilities one of which must happen, each has a 1/1:000...000=0:000...001 chance of being chosen.

Tony:

and yet
each has a chance of being chosen, and so
the probability is really nonzero.
It's only zero in standard finite analysis, but
where you are talking about an
infinite set of possibilities, you are outside of
standard territory, and the
answer is nonstandard: each possiblity has
an infinitesimal probability, and
that infinity of infinitesimal values sums to 1
as expected. So, n*1/n=1 holds
in the infinite case, and there is no reason to
expect it to fail. Is there?

Problem here is that you haven't really defined
"infinitesimal" in any rigorous
and non−circular way.

Immaterial. If n is infinite, then 1/n is infinitesimal. It's smaller than any finite value and yet nonzero, just like the probability we're discussing.

That's still circular. If you haven't defined infinitesimal carefully, and only

> in terms of other well−defined concepts (and "infinite" isn't such a concept, because you haven't really defined it well except in relation to infinitesimals), then you can't use the term to explain or define anything else.

If we are discussing an infinite set of possiblities, and I define infinitesimal in terms of that infinity, it's not circular. If you want to have a rigorous notion, start with infinity, but I rather doubt your "injections into a proper subset" definition is going to help a whole lot here. This isn't rocket science. n*1/n=1 and lim(n−>oo: n*1/n)=1 and lim(n−>oo:n)*lim(n−>oo: 1/n)=1. All is One, Cricket.

> > Charles said:
> >
> > > > Similarly, if the sum of all steps "should be" 2, that's what I get as I travel along the tiny stair steps. So, it's GOOD that n*2/n equals 2 for all (finite) n; in fact, I find it extremely good evidence that in the limit, it will be 2 as well.
> >
> > Tony:
> >
> > > If you construct a stairway there, well, it will be 2. That's not a diagonal line. So, fine, your derivation is correct. No matter what size stairs you use, or how many, it will always add up to 2. Nice proof. It just has nothing to do with a diagonal line. The sum of the probabilities of all mutually exclusive possibilities summing to 1, on the other hand, is a cornerstone of probability theory, and something that should be considered always true.
> >
> > If one can find a probability distribution for the elements, yes. Otherwise all bets are off (pun intended).
>
> Whether one can determine a distribution or not, there is never 110% chance

of
something happening. If you have a set of mutually exclusive possiblities, and
one of them is going to happen, the probabilities of each happening must sum to
1. If it sums to less than 1, then you have left out some possibliites, and if it sums to greater than 1, then some possiblities are not mutually exclusive.

Again, this is only true if there exists a probability distribution (determined or not) on the set. If there doesn't exist a probability distribution, and there seems to be disagreement on this point, then what you say is simply not the case.

We're no longer talking about the problematic set of naturals, as far as I'm concerned.

Charles:

If I travel along the stair steps, no matter how small, then sum of all the stair steps "should" sum to 2. The fact you seem to think that the distance in the infinite case is actually sqrt(2) indicates an error.
The sum is correctly 2 for all finite n. The sum is correctly 2 for infinite n as well, but the length of each stair step is 2/n, which in the infinite case is an infinitesimal hodon [*], not absolute 0.

Yeah, sure. Like I said, it's not the diagonal line, even though it starts to look like it. You're talking about a fractal dimension on the line, basically.
What does that have to do with probability? It's just an attempt to change the subject into something ridiculous and trying

to say it's the same argument.

Is the parallel making sense
to you yet? Do you see /any/
difference in
these /arguments/, besides
the fact that one gives an
answer you "want"
or "should get", whereas the
other doesn't?

Tony:

Yes, I do. The difference is that you're trying
to say your example is about a
diagonal line, and it's not, whereas Han's
trying to say his example is about
probability in an infinite set, which it is, to
whatever extent he actually
means "infinite". ;−)

Both Han's proof and Charles' depend on reasoning from the
behavior of an
increasing, but always finite, number of elements to an
infinite number of
elements. To that extent, the proofs expose the same types of
reasoning
flaws.

There's really no flaw in proving an equality between expressions in the infinite case. Inequalities can be invalid in the infinite case, where the difference causing the inequality has a limit of 0 as n−>oo. Here, the flaw was
simply that the staircase never was a diagonal line, even if it appears so visually as n−>oo. Chas mentioned approximating lengths of curves with infinitesimals, but those are always parallel to the curve, while his are always at a 45 degree angle, causing the sqrt(2) error by the sine and cosine of that angle.

Could you explain what it means for an infinitesimal (sc. "infinitesimally long line segment", I suppose) to be "parallel" to a curve?

Do you need this explained? It means that the segment ((x1, y1),(x2, y2)) is parallel to the curve if there can be defined a point on the curve perpendicular to a point on the segment where points (x3, y3) and (x4, y4) arbitrarily close to that point on either side have the property that (x3−x4)/(y3−y4)=(x1−x2)/(y1−y2), or at leats the difference is arbitrarily close to 0.

You'll probably have a correction regarding this, but then, why did you ask?
You know what parallel means, and it doesn't mean at a 45 degree angle.

Charles:

> * No one is claiming that for
> finite n, n*1/n equals
> something other
> than 1, anymore than than
> any one would claim that
> n*2/n equals
> something other than 2.

Tony:

Good, so no one's totally insane....

Charles:

> So there's no reason to
> repeat these assertions; they
> are accepted.
>
> Each of the claims you have
> made so far for the
> correctness of Han's
> argument can be mirrored as
> an equally valid claim for
> correctness of
> my argument; just substitute
> "2" for "1" and "sqrt(2)" for
> "0", and
> "finite number of stair steps"
> for "finite set of outcomes".

Tony:

Great, so it's true for the infinite staircase.
And?

Charles:

> But my /conclusion/ is so
> obviously false (by appeal

to Pythagoras),
there must be something
wrong with my /argument/.

Tony:

Yes, it has nothing to do with the diagonal
line. Your line is still vertical
and horizontal.

And Han's increasingly large finite sets are still finite,
whereas his
conclusion is drawn about something infinite. What's the
difference between
using finite sums of lengths to draw conclusions about an
"infinite" (in
some sense) sum of lengths, and using finite sets of
equiprobable elements
to draw conclusions about an infinite set of elements?

No difference whatsoever, and Chas' conclusion that the infinitely fine staircase still requires a line of length 2 is correct, but it's not a diagonal line. The only mistake is thinking it's a diagonal line. the length is 2, and the sum of the probabilities is 1, by proper inductive proof of equality holding for the infinite case. Bith proofs are valid in my opinion.

Neither is true, because you're using induction on finite quantities
to assert results about infinite quantities.

That is not a violation, as I see it, as long as one proves an equality.

I claim that if you can figure
out why the /argument/ is
wrong (not
just the conclusion), then
you will also see why Han's
argument is
wrong (above and beyond
the fact that his conclusion
is also
independently false,
although perhaps not as
obviously, by appeal to

Archimedes).

But it's not. Archimedean principle is mitigated at the infinitesimal level by the fact that neighboring infinitesimals do not constitute distinct standard reals, and so on the finite level, any discreteness of the infinitesimals, if they are taken to be discrete, does not violate Archimedean principle on that level. Further, on the infintiesimal level, Archimedean principle CAN be preserved by defining midpoints in the infinitesimal intervals, because at that level, the endpoint ARE distinct values in their nonstandard sense.

Again, you haven't defined infinitesimals rigorously and non−circularly,
so referring to them in an argument can't lead to valid conclusions.

Waaahhh...

That's how reasoning works. One may arrive at a true conclusion using ill−defined terms, but the argument leading to the conclusion will be an invalid one.

Like your ill defined diagonal as the limit of the staircase as n−>oo.

Smaller than any finite but non zero. The multiplicative inverse of an infinity.

That's circular, because you haven't come up with a coherent definition of infinity, nor any reason to believe that the inverse of "an infinity"
will be non−zero.

First of all, if you have an objection regarding the definition of infinity,
you should have brought that up weeks ago. Secondly, the argument regarding the chance of one of an infinite set being chosen being smaller than any finite value, and yet nonzero, has been exhaustively discussed without any valid objection. Each has a probability of 0% in the standard world, but is not

without a chance in reality. Been there, done that. Each of n equally likely events has probability 1/n, and if n is infinite, as put forth to begin with, this probability is infinitesimal.

Why is your conclusion wrong? It's not, if you're talking about a line broken into vertical and horizontal elements. It will always be equal to the sum of the vertical and horizontal distances traveled, because it's NOT the diagonal between the starting and ending points.

What gives you the idea that one can "sum" the probabilities of an infinite number of points to get the whole (1) any more than one can add up infinite numbers of vertical and horizontal lengths to get a diagonal length?

Because they add linearly, without the need to be parallel to what they "approximate", since the values are scalar and cannot be anything BUT parallel, as opposed to the vectors that the treads and risers represent. If all your infinitesimal segments were PARALLEL to the diagonal, then indeed they would sum to, guess what, sqrt(2)!

But how do you know that your probabilities add? Without a probability distribution (known or unknown), they don't.

Sum(x=1−>n: 1/n)=1. We already DEFINED the probability distribution to be uniform, not over the naturals, but from 1 through n.

If you have n possibilities

all mutually exclusive and equally likely, and one of them must occur, then the chances that any given one will occur is 1/n, so that the probabilities of all will sum to 1, as expected.

If you have n steps in the diagonal, the length of each tread/riser is 2/n, so that the total length will sum to 2, as expected.

So, you EXPECT the diagonal to be equal to 2? Whatever. You're not discussing things constructively at all.

This is much closer to the problem:

Why would you think that the answer is anything BUT 2? In fact, the proof shows it /must/ be 2!

Equally, why would the sum of a countable number of equal reals not possibly be 1? In fact, the proof shows it /must/ be 1!

I'm not especially comfortable with the use of the word "countable" in this context. We are not talking about a uniform distribution over the standard naturals. We've already agreed that doesn't exist, since it depends directly on the contradictory "largest finite". What Han is suggesting is that the sum of the probabilities of mutually exclusive events, one of which must happen, is equal to 1, and that this rule should be preserved in all cases, including the

infinite case. He is pointing out that standard
analysis fails in this regard,
and seems amenable to the notion that
infinitesimal probabilties solve the
problem. The question is, why aren't you?

See earlier comments on "infinitesimal". I'm still awaiting
the axiomatization
of "infinite" and "infinitesimal" which you said in another
thread you were
working on. Until I can see that, the answer to your question
must be "Because
I don't know that it makes sense."

How on Earth can it be that n*1/n<>1? How can that make sense?

It doesn't make sense, if n exists and if one can do ordinary arithmetic with n.
What I'm saying is that, absent an axiomatization such as I mentioned, I don't
know that either of these premises is true.

And you don't know they're not. You have certainly heard enough
"circumstantial" evidence that points to a strong possiblity, but your
assessment of that possibility appears to be under the influence of other
factors.

<snip>

My argument claims that the
right value is 2, and
that sqrt(2) is wrong (and
therefore Euclidean
geometry is flawed). Han
claims the right value is 1,
and that 0 is wrong (and
therefore, the
standard reals are flawed).
We both maintain our
respective right
values, all the way out to the
limit; keeping continuity
between the
finite and the "potential
infinite".

> > > Oh, that's a belch of hot air, Chas. How can
> > > you deny that the laws of
> > > probability define 1 as the probability of
> > > something that is definitely true?
> > > If we are choosing 1 of a set of elements, the
> > > probability that one from that
> > > set will be chosen is 1. Do you disagree yet?
> >
> > I don't know about him, but I don't. Yet.
>
> > > Can you deny that the laws of probability
> > > say that the probability of one of a
> > > set of mutually exclusive possibilities
> > > happening is equal to the sum of each
> > > of them happening?
> >
> > As phrased, yes, I can and do deny it.
>
> What in the world could you be objecting to? Say we have a set of 3 mutually
> exclusive possibilities, A, B and C. Is the probability P(A or B or C) not
> equal to P(A)+P(B)+P(C)? At least in the finite case, you must agree.

That's because, in the finite case, one can always arrive at a probability
distribution over the set.

And if one postulates an infinite set with a uniform probability distribution,
can one assume that it has a uniform probability distribution? (sigh)

> > > Do you disagree that if there is a uniform
> > > probability distribution, that each
> > > possiblity has an equal probability?
> >
> > No, I don't disagree. But I thought that we were agreed there
> > was no probability
> > distribution?
>
> Forget the naturals already. That wasn't Han's original idea. Set theorists
> brought that in to confound the topic. Drop it.

It's not the naturals; it's any infinite set.

If n is infinite, and there are n equally likely events, and one is going to happen, how can you argue there is no uniform probability distribution? There is by definition.

> > > Do you agree that there may be an infinite number of possiblities in some cases?
> >
> > Yes.
> >
> > > Does it make sense to you that in an infinite set of possibilities, at least SOME of them must have a probability of 0, even if they have a chance of occurring? Does this 0 mean "no chance at all", or is it simply 0 for lack of a standard treatment of the infinitesimals?
> >
> > It's not 0, as I understand it. It's simply that there's no well−defined way to talk about a number having "a chance of occurring". I don't have a problem with that. Should I?
>
> You can accept that as alright for your purposes, but that doesn't mean it's wrong to consider. If you don't care about how to talk about it, then don't. But don't sit around telling people they are being dumb for discussing it. Infinitesimal probabilities are the way to discuss it. If you don't like that, don't discuss it. Personally, I think it's crucial for the next step in the study of infinity and the development of the foundations of math. So, if you don't mind, I think I shall engage in such conversation to see what actual problems it causes. So far, I don't see any.

> Infinitesimal probabilities are dependent on the existence and putative nature of infinitesimals. So far, I don't see that you've developed an explicit theory governing their existence and nature.

Is that grounds to dismiss the notion entirely? It's obvious that in the case Han suggests, the individual probabilities are indeed infinitesimal, whether you have a formal system of representation, calculation, or anything else for those infinitesimal values. But still, don't forget the T−riffics.

I'm not saying that you're dumb for discussing it. I'm saying that without a rigorous understanding of your concepts of "infinite" and "infinitesimal," it's simply impossible to draw any certain conclusions about these sorts of probabilities. And I do see a problem with attempting to draw conclusions before you know that your arguments are valid.

These arguments are so basic, there is no reasonable doubt in my mind that what we are discussing is entire reasonable. It is clear that extensions to digital number systems can handle such cases, and that this is the only solution to the question. So, I dunno, I feel rather confident that Han is being argued with over nothing more than the suggestion that standards need to be expanded.

<snip>

Consider: isn't this "exactly how" how we are taught that the length of a continuous curve can be approximated by "infinitesimally small" line segments?

That doesn't work if you use infinitely small line segements that aren't PARALLEL with the curve you are approximating, now, does it? Your example is diagonal−free. Move on. Consider the fact that n*1/n=1 for specific infinite n, and its specific infinitesimal multiplicative inverse.

See above comments on infinitesimals.

Yeah, I saw.

"I don't know what you mean by 1/n being infinitesimal for infinite n because you haven't rigorously developed a theory I can microscopically analyze, and I can't understand simple concepts like lim(x−>oo: 1/x)−>0."

> If you haven't developed a theory that I can analyze (or any theory even vaguely formal), then I'm perfectly justified in questioning any conclusions you claim to be based on concepts of that "theory". And I can understand simple concepts like "the limit as x increases without bound of 1/x is zero"; I just don't see where infinity comes into the sentence.
>
> Matt

Yes, well, with the concerted effort in the last cetury plus to all but eliminate "infinity" from the vocabulary and compromise on a system the obfuscates the whole notion in favor of finite "rigor", it's not surprising. But, I'll remind you that "infinity" came into the "sentence" when Han postulated an infinite set with a uniform probability distribution.

--
Smiles,

Tony
.