Re: Calculus XOR Probability

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- *From*: Matt Gutting <tchrmatt@xxxxxxxxx>
- *Date*: Thu, 13 Apr 2006 08:03:48 −0400

Tony Orlow wrote:

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> > > > > cbrown@xxxxxxxxxxxxxxxxx said:
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> > > > <snip>
> > > >
> > > > > > Han seems unbothered by the fact that 1 is not the value you get from the sum of a countable number of 0's; he simply "wants" it to be 1. And lo and behold, that is what his argument affirms, just as mine affirms that the length of

the diagonal
is exactly
what I
"want" it to
be: 2.

Tony:

No, he doesn't just "want" it to be 1. That is the probability of something which is definitely true. If one of the elements is to be chosen from the set, then the probability of one being chosen is 1. If there are multiple choices, then that 1 representing the fact that one of those will be chosen is defined to be the sum of the probabilities of each. That's how probability works,

Only if you can define a probability distribution, which is the assumption in question here.

Not true. If you can establish a uniform probability distribution, then you can say each possibility has the same probability. If you can formulate the distribution at all, it will allow you to calculate which fraction of the overall probability is assigned to each possiblity. But whether you can determine what the distribution is or not, the overall probability that one will be chosen, given the fact that you're choosing one, is 1, and the probabilities of each mutually exclusive alternative contributes to that sum equal to 1. That's a very basic concept that must hold, even if we don't know what probability distribution we're looking at.

It must hold, even if we don't know what probability distribution we're talking
about, *assuming* that we're looking at a probability distribution. As I said, the question is whether one can be defined here. You appear to be assuming it
can. I'm not.

Somethign tells me you're still trying to deal with a uniform probability distribution over tha naturals. That's impossible, but given any set of n equally likely outcomes, if exactly one of them will occur, each has a 1/n chance of being the one to occur. To assume you have a set of

> possibilities WITHOUT a probability distribution is just to say you have no idea what any of the individual probabilities is. That has no effect on the fact that if exactly one is to occur, the probabilities of each sum to that 1.

I'm not assuming a *uniform* probability distribution over the naturals. I'm saying that what you are asserting is only true when one can define *some* probability distribution over the naturals. In effect, I'm not saying that I "have no idea what any of the individual probabilities is"; instead, I'm saying that referring to "individual probabilities" is meaningless in this case, so that saying "the individual probabilities sum to 1" is likewise meaningless.

> > The difficulty here is that you haven't really shown an explicit and consistent way of defining and working with infinitesimals. That must come first, before
> > you begin using them to explain things.

> What do you want, a number system? By all means, use the T−riffics. If we have 1:000...000 equally likely possibilities one of which must happen, each has a 1/1:000...000=0:000...001 chance of being chosen.

After reading through numerous posts on the subject, I still don't see that this is a consistent treatment of infinitesimals.

<snip>

> > > Immaterial. If n is infinite, then 1/n is infinitesimal. It's smaller than any finite value and yet nonzero, just like the probability we're discussing.

> > That's still circular. If you haven't defined infinitesimal carefully, and only in terms of other well−defined concepts (and "infinite" isn't such a concept, because you haven't really defined it well except in relation to infinitesimals), then you can't use the term to explain or define anything else.

> If we are discussing an infinite set of possiblities, and I define infinitesimal in terms of that infinity, it's not circular. If you want to have a rigorous notion, start with infinity, but I rather doubt your "injections into a proper subset" definition is going to help a whole lot here. This isn't rocket science. n*1/n=1 and lim(n−>oo: n*1/n)=1 and lim(n−>oo:n)*lim(n−>oo: 1/n)=1. All is One, Cricket.

The equation (lim x−>:a f(x)g(x)) = (lim x−>a:f(x))(lim x−>a:g(x)) is only true when both limits exist. It's explicitly proved that way. You can't use that theorem here, since (lim n−>oo:n) does not exist.

> > > Whether one can determine a distribution or not, there is never 110% chance of something happening. If you have a set of mutually exclusive possiblities, and one of them is going to happen, the probabilities of each happening must sum to 1. If it sums to less than 1, then you have left out some possibliites, and if it sums to greater than 1, then some possiblities are not mutually exclusive.
>
> > Again, this is only true if there exists a probability distribution (determined or not) on the set. If there doesn't exist a probability distribution, and there seems to be disagreement on this point, then what you say is simply not the case.
>
> We're no longer talking about the problematic set of naturals, as far as I'm concerned.

It doesn't matter what set we're talking about, as long as it's infinite (or,
if you prefer, unbounded).

<snip>

> > > There's really no flaw in proving an equality between expressions in the infinite case. Inequalities can be invalid in the infinite case, where the difference causing the inequality has a limit of 0 as n−>oo. Here, the flaw was simply that the staircase never was a diagonal line, even if it appears so visually as n−>oo. Chas mentioned approximating lengths of curves with infinitesimals, but those are always parallel to the curve, while his are always at a 45 degree angle, causing the sqrt(2) error by the sine and cosine of that angle.
>
> > Could you explain what it means for an infinitesimal (sc. "infinitesimally long line segment", I suppose) to be "parallel" to a curve?
>
> Do you need this explained? It means that the segment ((x1, y1),(x2, y2)) is parallel to the curve if there can be defined a point on the curve perpendicular to a point on the segment where points (x3, y3) and (x4, y4)
> arbitrarily close to that point on either side have the property that (x3−x4)/(y3−y4)=(x1−x2)/(y1−y2), or at leats the difference is arbitrarily close to 0. You'll probably have a correction regarding this, but then, why did you ask? You know what parallel means, and it doesn't mean at a 45 degree angle.

You're being, at best, very loose with language here: one can't have a point (on the curve or anywhere else) perpendicular to another point (on the segment or anywhere else). I *think* you mean that one can find a secant line "nearby" the curve (and yes, I'm aware that I'm using that loosely; I can formalize it if you'd like) which is parallel to a secant line on the other curve.

<snip>

> > > And Han's increasingly large finite sets are still finite, whereas his conclusion is drawn about something infinite. What's the difference between using finite sums of lengths to draw conclusions about an "infinite" (in some sense) sum of lengths, and using finite sets of equiprobable elements to draw conclusions about an infinite set of elements?
> >
> > No difference whatsoever, and Chas' conclusion that the infinitely fine staircase still requires a line of length 2 is correct, but it's not a diagonal line. The only mistake is thinking it's a diagonal line. the length is 2, and the sum of the probabilities is 1, by proper inductive proof of equality holding for the infinite case. Bith proofs are valid in my opinion.
>
> Neither is true, because you're using induction on finite quantities to assert results about infinite quantities.

That is not a violation, as I see it, as long as one proves an equality.

See above. You appear to be misunderstanding the limits of inductive arguments.

I claim that
if you can
figure out
why the
/argument/
is wrong
(not
just the
conclusion),
then you

> > > > will also see why Han's argument is wrong (above and beyond the fact that his conclusion is also independently false, although perhaps not as obviously, by appeal to Archimedes).
> > >
> > > But it's not. Archimedean principle is mitigated at the infinitesimal level by the fact that neighboring infinitesimals do not constitute distinct standard reals, and so on the finite level, any discreteness of the infinitesimals, if they are taken to be discrete, does not violate Archimedean principle on that level. Further, on the infintiesimal level, Archimedean principle CAN be preserved by defining midpoints in the infinitesimal intervals, because at that level, the endpoint ARE distinct values in their nonstandard sense.
> >
> > Again, you haven't defined infinitesimals rigorously and non−circularly, so referring to them in an argument can't lead to valid conclusions.
>
> Waaahhh...

That's how reasoning works. One may arrive at a true conclusion using ill−defined terms, but the argument leading to the conclusion will be an invalid one.

Like your ill defined diagonal as the limit of the staircase as n−>oo.

That's exactly the point. That argument was intentionally posed as a flawed argument, to point out the flaws in Han's argument.

Smaller than any finite but non zero. The multiplicative inverse of an infinity.

That's circular, because you haven't come up with a coherent definition of infinity, nor any reason to believe that the inverse of "an infinity" will be non−zero.

First of all, if you have an objection regarding the definition of infinity, you should have brought that up weeks ago. Secondly, the argument regarding the chance of one of an infinite set being chosen being smaller than any finite value, and yet nonzero, has been exhaustively discussed without any valid objection. Each has a probability of 0% in the standard world, but is not without a chance in reality. Been there, done that. Each of n equally likely events has probability 1/n, and if n is infinite, as put forth to begin with, this probability is infinitesimal.

I believe I did bring up objections regarding your usage of infinity; we have discussed elsewhere the need for you to formalize and axiomatize your notions of infinity, and you agreed that it was necessary and important. I haven't seen any development in this area.

The fact that you have seen no valid objection to an argument is not necessarily an indicator that there is no valid objection; it just means that you haven't seen one. As I said above, the assumption you're making here (one of the assumptions, at any rate) is that it makes sense, when talking about infinite sets, to consider the phrase "equally likely" as meaningful. It's not.

Why is your conclusion wrong? It's not, if you're talking about a line broken into vertical and horizontal elements. It will always be equal to the sum of the vertical and horizontal distances traveled, because it's NOT the diagonal between the starting and ending points.

> > > > What gives you the idea that one can "sum" the probabilities of an infinite number of points to get the whole (1) any more than one can add up infinite numbers of vertical and horizontal lengths to get a diagonal length?
> > >
> > > Because they add linearly, without the need to be parallel to what they "approximate", since the values are scalar and cannot be anything BUT parallel, as opposed to the vectors that the treads and risers represent. If all your infinitesimal segments were PARALLEL to the diagonal, then indeed they would sum to, guess what, sqrt(2)!
> >
> > But how do you know that your probabilities add? Without a probability distribution (known or unknown), they don't.
>
> Sum(x=1−>n: 1/n)=1. We already DEFINED the probability distribution to be uniform, not over the naturals, but from 1 through n.

That works when n is finite; that is, it works for any set 1 through n. But none of these sets is the natural numbers. If you're not talking about an infinite set, we have no argument. But if you are, then what infinite set are you intending to discuss? The same objection holds for any infinite set: the concept of "probability of an individual event occurring" is meaningless in such a context.

<snip>

> > > How on Earth can it be that n*1/n<>1? How can that make sense?
> >
> > It doesn't make sense, if n exists and if one can do ordinary arithmetic with n. What I'm saying is that, absent an axiomatization such as I mentioned, I don't know that either of these premises is true.
>
> And you don't know they're not. You have certainly heard enough "circumstantial" evidence that points to a strong possiblity, but your assessment of that possibility appears to be under the influence of other factors.

If you don't have an axiomatization, it is meaningless to say that they are true, just as it is meaningless to say that they are false.

<snip>

> My argument claims that the right value is 2, and that sqrt(2) is wrong (and therefore Euclidean geometry is flawed). Han claims the right value is 1, and that 0 is wrong (and therefore, the standard reals are flawed). We both maintain our respective right values, all the way out to the limit; keeping continuity between the finite and the "potential infinite".

Oh, that's a belch of hot air, Chas. How can you deny that the laws of probability define 1 as the probability of something that is definitely true? If we are choosing 1 of a set of elements, the probability that one from that set will be chosen is 1.

Do you disagree yet?

I don't know about him, but I don't. Yet.

Can you deny that the laws of probability say that the probability of one of a set of mutually exclusive possibilities happening is equal to the sum of each of them happening?

As phrased, yes, I can and do deny it.

What in the world could you be objecting to? Say we have a set of 3 mutually exclusive possibilities, A, B and C. Is the probability P(A or B or C) not equal to P(A)+P(B)+P(C)? At least in the finite case, you must agree.

That's because, in the finite case, one can always arrive at a probability distribution over the set.

And if one postulates an infinite set with a uniform probability distribution, can one assume that it has a uniform probability distribution? (sigh)

Only if it turns out to be logically consistent for an infinite set to have a uniform probability distribution. You can't work with sets of postulates that are logically inconsistent.

Do you disagree that if there is a uniform probability distribution, that each possiblity has an equal probability?

No, I don't disagree. But I thought that we were agreed there was no probability distribution?

Forget the naturals already. That wasn't Han's original idea. Set theorists brought that in to confound the topic. Drop it.

It's not the naturals; it's any infinite set.

> If n is infinite, and there are n equally likely events, and one is going to happen, how can you argue there is no uniform probability distribution? There is by definition.

Again, this depends on the phrase "equally likely events" having meaning in this context. I contend that there is no logically consistent meaning that can be assigned to the phrase in this context.

<snip>

>> Infinitesimal probabilities are dependent on the existence and putative nature of infinitesimals. So far, I don't see that you've developed an explicit theory governing their existence and nature.

> Is that grounds to dismiss the notion entirely? It's obvious that in the case Han suggests, the individual probabilities are indeed infinitesimal, whether you have a formal system of representation, calculation, or anything else for those infinitesimal values. But still, don't forget the T−riffics.

Nothing is "obvious". (I noticed that most of all in math textbook proofs. Whenever they said "it is clear/obvious/trivial that..." I knew I was in for the hard work trying to figure out *why* it was so.) I'm not worried about a formal system of representing or calculating with them. I'm worried about a formal system of *defining* them: as you've mentioned elsewhere, you're apparently working on one, and you seemed to have accepted my statement that until you did so, I couldn't accept any statement you made about them.

>> I'm not saying that you're dumb for discussing it. I'm saying that without a rigorous understanding of your concepts of "infinite" and "infinitesimal," it's simply impossible to draw any certain conclusions about these sorts of probabilities. And I do see a problem with attempting to draw conclusions before you know that your arguments are valid.

> These arguments are so basic, there is no reasonable doubt in my mind that what we are discussing is entire reasonable. It is clear that extensions to digital number systems can handle such cases, and that this is the only solution to the question. So, I dunno, I feel rather confident that Han is being argued with over nothing more than the suggestion that standards need to be expanded.

Again, this is not "clear" to me, nor to others.

> > > "I don't know what you mean by 1/n being infinitesimal for
> > > infinite n because you haven't rigorously developed a theory
> > > I can microscopically analyze, and I can't understand simple
> > > concepts like lim(x−>oo: 1/x)−>0."
> >
> > If you haven't developed a theory that I can analyze (or any theory even
> > vaguely
> > formal), then I'm perfectly justified in questioning any conclusions you claim
> > to be based on concepts of that "theory". And I can understand simple
> > concepts
> > like "the limit as x increases without bound of 1/x is zero"; I just don't see
> > where infinity comes into the sentence.
> >
> > Matt
>
> Yes, well, with the concerted effort in the last cetury plus to all but eliminate "infinity" from the vocabulary and compromise on a system the obfuscates the whole notion in favor of finite "rigor", it's not surprising. But, I'll remind you that "infinity" came into the "sentence" when Han postulated an infinite set with a uniform probability distribution.

And he based his conclusions on analogies to finite cases, thus eliminating
"infinity" from the sentence again. The counter−arguments were simply that
it isn't valid to reason from those finite cases to infinite ones. Those
arguments still stand.

Matt
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