Re: Calculus XOR Probability

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- *From*: Matt Gutting <tchrmatt@xxxxxxxxx>
- *Date*: Thu, 13 Apr 2006 15:08:54 −0400

Tony Orlow wrote:

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>>>>>> Tony Orlow wrote:
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>>>>>>> cbrown@xxxxxxxxxxxxxxxxx said:
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>>>>>> <snip>
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>>>>>>>> Han seems unbothered by the fact that 1 is not the value you get from the sum of

a countable number of 0's; he simply "wants" it to be 1. And lo and behold, that is what his argument affirms, just as mine affirms that the length of the diagonal is exactly what I "want" it to be: 2.

Tony:

No, he doesn't just "want" it to

be
1.
That
is
the
probability
of
something
which
is
definitely
true.
If
one
of
the
elements
is
to
be
chosen
from
the
set,
then
the
probability
of
one
being
chosen
is
1.
If
there
are
multiple
choices,
then
that
1
representing
the
fact
that
one
of
those
will
be
chosen

is defined to be the sum of the probabilities of each. That's how probability works,

Only if you can define a probability distribution, which is the assumption in question here.

Not true. If you can establish a uniform probability distribution, then you can say each possibility has the same probability. If you can formulate the distribution at all, it will allow you to calculate which fraction of the overall probability is assigned to each possiblity. But whether you can determine what the distribution is or not, the overall probability that one will be chosen, given the fact that you're choosing one, is 1, and the probabilities of each mutually exclusive alternative contributes to that sum equal to 1. That's a very basic concept that must hold, even if we don't know what probability distribution we're looking at.

It must hold, even if we don't know what probability distribution we're talking about, *assuming* that we're looking at a probability distribution. As I said, the question is whether one can be defined here. You appear to be assuming it can. I'm not.

Somethign tells me you're still trying to deal with a uniform probability distribution over tha naturals. That's impossible, but given any set of n equally likely outcomes, if exactly one of them will occur, each has a 1/n chance of being the one to occur. To assume you have a set of possibilities WITHOUT a probability distribution is just to say you have no idea what any of the individual probabilities is. That has no effect on the fact that if exactly one is to occur, the probabilities of each sum to that 1.

I'm not assuming a *uniform* probability distribution over the naturals. I'm saying that what you are asserting is only true when one can define *some* probability distribution over the naturals. In effect, I'm not saying that I "have no idea what any of the individual probabilities is"; instead, I'm saying that referring to "individual probabilities" is meaningless in this case, so that saying "the individual probabilities sum to 1" is likewise meaningless.

First of all, discussing the naturals is a digression and a straw man. We have n, the upper bound of the set. There are n elements. The probability of each being chosen has meaning if we are choosing one. If we know how many we have, if we can calculate and average of n/2, then we can assign a probability, albeit infinitesimal. With the naturals, you can't. We agree. Forget the naturals.

If you have finitely many elements, then I'm fine with what you're saying.
But if you have infinitely many, then there's a problem.

The difficulty here is that you haven't really shown an explicit and consistent way of defining and working with infinitesimals. That must come first, before you begin using them to explain things.

What do you want, a number system? By all means, use the T−riffics. If we have 1:000...000 equally likely possibilities one of which must happen, each has a 1/1:000...000=0:000...001 chance of being chosen.

After reading through numerous posts on the subject, I still don't see that this is a consistent treatment of infinitesimals.

What inconsistencies do you detect?

There are a bunch of issues with "scale" and "sub−infinitesimal numbers" that don't seem to fit with the way order relationships work on the real numbers.

<snip>

Immaterial. If n is infinite, then 1/n is infinitesimal. It's smaller than any finite value and yet nonzero, just like the probability we're discussing.

That's still circular. If you haven't defined infinitesimal carefully, and only in terms of other well−defined concepts (and "infinite" isn't such a concept, because you haven't really defined it well except in relation to infinitesimals), then you can't use the term to explain or define anything else.

If we are discussing an infinite set of possiblities, and I define infinitesimal in terms of that infinity, it's not circular. If you want to have a rigorous notion, start with infinity, but I rather doubt your "injections into a proper subset" definition is going to help a whole lot here. This isn't rocket science. n*1/n=1 and lim(n−>oo: n*1/n)=1 and lim(n−>oo:n)*lim(n−>oo: 1/n)=1. All is One, Cricket.

The equation (lim x−>:a f(x)g(x)) = (lim x−>a:f(x))(lim x−>a:g(x)) is only true
when both limits exist. It's explicitly proved that way. You can't use that theorem here, since (lim n−>oo:n) does not exist.

Sure, I know, "undefined". So far, undefined. Undefinable? Is sqrt(−1) undefinable? It used to be undefined. What happened? Did we define it and find it useful? Gee, could that happen here? Sure, that's what's going on. (lim n−>oo:n)=oo. Now, if you used a SPECIFIC oo, you'd have a specific answer, wouldn't you?

I don't know that many (or any, for that matter) other people know what you mean when you say "a SPECIFIC oo". The problem goes back, again, to your insistence that there exist infinitely large natural numbers. There don't, and defining them into existence puts them in conflict with basic axioms defining and describing numbers. You haven't worked out any of these conflicts (at least, I haven't seen a consistent list of axioms), and even if you had, you would be working in a number system entirely different from the standard, thus making your conclusions inapplicable to what we're talking about.

> > > > > Whether one can determine a distribution or not, there is never 110% chance of something happening. If you have a set of mutually exclusive possiblities, and one of them is going to happen, the probabilities of each happening must sum to 1. If it sums to less than 1, then you have left out some possibliites, and if it sums to greater than 1, then some possiblities are not mutually exclusive.

> > > > Again, this is only true if there exists a probability distribution (determined or not) on the set. If there doesn't exist a probability distribution, and there seems to be disagreement on this point, then what you say is simply not the case.

> > > We're no longer talking about the problematic set of naturals, as far as I'm concerned.

> > It doesn't matter what set we're talking about, as long as it's infinite (or, if you prefer, unbounded).

> No, it must be infinite, yet bounded, to be able to define the probability of an individual element. The problem with the naturals is a lack of defined range, which of course has been a sticky issue when I've been trying to put forth the Inverse Function Rule.

No, the problem with the naturals is an infinite amount of elements.

<snip>

There's really no flaw in proving an equality between expressions in the infinite case. Inequalities can be invalid in the infinite case, where the difference causing the inequality has a limit of 0 as n−>oo. Here, the flaw was simply that the staircase never was a diagonal line, even if it appears so visually as n−>oo. Chas mentioned approximating lengths of curves with infinitesimals, but those are always parallel to the curve, while his are always at a 45 degree angle, causing the sqrt(2) error by the sine and cosine of that angle.

Could you explain what it means for an infinitesimal (sc. "infinitesimally long line segment", I suppose) to be "parallel" to a curve?

Do you need this explained? It means that the segment ((x1, y1),(x2, y2)) is parallel to the curve if there can be defined a point on the curve perpendicular to a point on the segment where points (x3, y3) and (x4, y4)
arbitrarily close to that point on either side have the property that (x3−x4)/
(y3−y4)=(x1−x2)/(y1−y2), or at leats the difference is arbitrarily close to 0. You'll probably have a correction regarding this, but then, why did you ask? You know what parallel means, and it doesn't mean at a 45 degree angle.

You're being, at best, very loose with language here: one can't have a point (on the curve or anywhere else) perpendicular to another point (on the segment
or anywhere else). I *think* you mean that one can find a secant line "nearby"
the curve (and yes, I'm aware that I'm using that loosely; I can formalize

it if you'd like) which is parallel to a secant line on the other curve.

Sure, whatever. Do you see that the angle at which your elements intersect the diagonal directly accounts for the error of sqrt(2)? If not, I went into greater detail in an earlier post to Chas.

<snip>

And Han's increasingly large finite sets are still finite, whereas his conclusion is drawn about something infinite. What's the difference between using finite sums of lengths to draw conclusions about an "infinite" (in some sense) sum of lengths, and using finite sets of equiprobable elements to draw conclusions about an infinite set of elements?

No difference whatsoever, and Chas' conclusion that the infinitely fine staircase still requires a line of length

2 is correct, but it's not a diagonal line. The only mistake is thinking it's a diagonal line. the length is 2, and the sum of the probabilities is 1, by proper inductive proof of equality holding for the infinite case. Bith proofs are valid in my opinion.

Neither is true, because you're using induction on finite quantities
to assert results about infinite quantities.

That is not a violation, as I see it, as long as one proves an equality.

See above. You appear to be misunderstanding the limits of inductive arguments.

I appear to be of a different opinion regarding this matter. You can chalk it up to misunderstanding if that makes you feel better, but my arguments in this area have all held, as far as I've been able to tell. COunterxamples like this are easily explained, so my point remains unrefuted.

I disagree. I haven't seen any counterexamples that you've adequately refuted.

I claim that if you can figure out why the /argument/ is wrong (not just the conclusion), then you will also see why Han's argument is wrong (above and beyond the fact that his conclusion is also independently false, although perhaps not as obviously, by appeal to Archimedes).

But it's not. Archimedean principle is mitigated at the infinitesimal level by the fact that neighboring infinitesimals do not constitute distinct standard reals, and so on the finite level, any discreteness of the infinitesimals, if they are taken to be discrete, does not violate Archimedean principle on that level. Further, on the infintiesimal level, Archimedean principle CAN be preserved by defining midpoints in the infinitesimal intervals, because at that level, the

endpoint
ARE
distinct
values
in
their
nonstandard
sense.

Again, you
haven't
defined
infinitesimals
rigorously
and
non−circularly,
so referring
to them in
an argument
can't lead to
valid
conclusions.

Waaahhh...

That's how reasoning works. One may arrive
at a true conclusion using
ill−defined terms, but the argument leading
to the conclusion will be an
invalid one.

Like your ill defined diagonal as the limit of the staircase as
n−>oo.

That's exactly the point. That argument was intentionally posed as a flawed
argument, to point out the flaws in Han's argument.

But, the flaw in that argument is not what makes it similar to han's argument. The only flaw is
in equating the fractal diagonal with the normal self−parallel diagonal. They're different
animals, despite the fact that your microscope is too feeble to see the difference.

The set of points in this "limiting staircase" is precisely the diagonal.
Charles has in fact proved this.

Smaller than any finite but
non zero. The multiplicative

>>>> inverse of an infinity.
>>>
>>> That's circular, because you haven't come up
>>> with a coherent definition of
>>> infinity, nor any reason to believe that the
>>> inverse of "an infinity"
>>> will be non−zero.
>>
>> First of all, if you have an objection regarding the definition of infinity, you should have brought that up weeks ago. Secondly, the argument regarding the chance of one of an infinite set being chosen being smaller than any finite value, and yet nonzero, has been exhaustively discussed without any valid objection. Each has a probability of 0% in the standard world, but is not without a chance in reality. Been there, done that. Each of n equally likely events has probability 1/n, and if n is infinite, as put forth to begin with, this probability is infinitesimal.
>
> I believe I did bring up objections regarding your usage of infinity; we have discussed elsewhere the need for you to formalize and axiomatize your notions
> of infinity, and you agreed that it was necessary and important. I haven't seen any development in this area.

I haven't been doing that all over the newsgroup. Just getting basic notions like the fractal natural of Chas' line is difficult enough. When I state a rule clearly, I get complaints about every term. I think the theory ends up putting forth infinite sets in the context of order, such that:

x<y<z −> x<z Order
x<z −> x<y<z Internal Infinity
y −> x<y<z External Infinity

I honestly don't know what this means. This may be why you "get complaints about every term". You abbreviate so much that it's difficult to tell what you mean, and when you don't abbreviate, you use terms in senses that you don't always make clear. Until you can do this, I can't accept any arguments you make based on your terminology.

I gave a bunch of this to Tribble a couple days ago, and went into discussion of how both use the same formula for generating elements in opposite directions, at least in one sense. That needs to be expanded to include another interpretation of successor that produces the H−riffics. In any case, can you think of a sense of infinity where a nonzero reciprocal could be finite, or anything but infinitesimal?

> The fact that you have seen no valid objection to an argument is not necessarily an indicator that there is no valid objection; it just means that you haven't seen one. As I said above, the assumption you're making here (one
> of the assumptions, at any rate) is that it makes sense, when talking about infinite sets, to consider the phrase "equally likely" as meaningful. It's not.

Why not? It's impossible to define a uniform probability distribution on the naturals due to its unboundedness, but given an infinite upper bound of n, it's not impossible to define the likelihood of individual elements. It's just not part of standard mathematics.

What do you mean, "an infinite upper bound"?

Why is your conclusion wrong? It's not, if you're talking about a line broken into vertical and horizontal elements. It will always be equal to the sum of the vertical and horizontal distances traveled, because it's NOT the diagonal between the starting and ending points.

What gives you the idea that one can "sum" the probabilities of an infinite number of points to get the whole (1) any more than one can add up infinite numbers of vertical and horizontal lengths to get a diagonal length?

Because they add linearly, without the need to be parallel to what they "approximate", since the values are scalar and cannot be anything BUT parallel, as opposed to the vectors that the treads and risers represent. If all your infinitesimal segments were PARALLEL to the diagonal, then indeed they would sum to, guess what, sqrt(2)!

But how do you know that your probabilities add? Without a probability distribution (known or unknown), they don't.

Sum(x=1−>n: 1/n)=1. We already DEFINED the probability distribution to be uniform, not over the naturals, but from 1 through n.

That works when n is finite; that is, it works for any set 1 through n.

It works for any set from 1 through n, whether n is finite or infinite.

But is there an infinite n? I think that's part of the problem.

But none of these sets is the natural numbers.

Good. To hell with the naturals. They make a crappy domain.

If you're not talking about an infinite
set, we have no argument. But if you are, then what infinite set are you intending to discuss? The same objection holds for any infinite set: the concept of "probability of an individual event occurring" is meaningless in such a context.

That's opinion, not logic. If you throw a dart at a board randomly, every point has a chance of being hit, but there are an infinity of points, so that chance is effectively 0, but not really 0. Is it meaningless to talk about the chance of a single point being hit? That idea has meaning for me.

The problem is that the consequences of the idea lead to contradictions which can only be resolved by dealing much more rigorously than you have with concepts of infinity and infinitesimals. The idea of something being "effectively 0, but not really 0" has no meaning for me.

Flip a coin aleph_1 times and generate bits to find a point in [0,1). Is there not a uniform probability distribution among the points? Throw your balls all in a vase by noon, shake vigorously an infinite number of times, and pick. Is one ball more likely than any other?

No, nor is it "as likely as any other" − the concept of "likelihood" needn't apply here.

Matt

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