

zorn's lemma and ascending chain condition

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I am trying to figure out if the proof of the following well known fact uses Zorn's lemma:

Let V be a partially ordered set. Suppose V satisfies the ascending chain condition (every strictly increasing sequence $x_1 < x_2 < x_3 < \dots$ is finite). Show that every non-empty subset of V has a maximal element.

The usual proof goes as follows: assume there is a nonempty subset T that has no maximal element, then there exists element x_1 in T since T is non empty, and then there exists bigger element x_2 in T since x_1 is not maximal, and then an x_3 in T such that $x_1 < x_2 < x_3$, and continue in this manner. We get an infinite strictly increasing sequence, a contradiction.

Am I using Zorn's lemma / axiom of choice here? It seems that in constructing the sequence I'm making infinitely many arbitrary choices, but I'm not sure.

Thanks,
Mike

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