gradient field/geodesics

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Is there anyone who could give me please help on the question:

Let M be a Riemannian manifold and let f be a smooth function on M whose gradient vector field G=grad(f) has |G|=1 everywhere. Show that the integral curves of G are geodesics.

This is easy to prove in R^n with the standard metric, because there the equation <G,G>=1 is equivalent

sum f_{x_i}^2 =1, (where f_{x_i} denotes the ith partial derivative)

and we differentiate this equation with respect to each variable respectively to see that the covaraint derivative of G with respect to itself is zero.

However in more general settings I cannot see how to do it; the same method hasn't worked for me.

Some help would really make my day.

Thanks

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