Re: Question

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In article <1146156962.005648.209040@xxxxxxxxxxxxxxxxxxxxxxxxxxxxx>,
zuhair <zaljohar@xxxxxxxxx> wrote:

[.snip.]

> Alternatively, one can define a "distance" of rational numbers by the usual means: the distance between a/b and c/d is |ad−bc|/bd, where |ad−bc| is the absolute value of ad−bc.
>
> A sequence of rationals is function from the natural numbers to the rationals.
>
> We say a sequence (a_0, a_1,...) (usually denoted {a_i}) converges to the rational number Q if and only for every N>0 there exists M>0 such that if n>M, then |a_n−Q|<1/N.
>
> We say a sequence (a_0, a_1, ...) is a "Cauchy sequence" if and only if for every N>0 there exists M>0 such that if n,m>M, then |a_n−a_m|<1/N.
>
> It is easy to verify that if a sequence converges to some rational, then it is Cauchy, though the converse does not hold.
>
> We can define an equivalence relation among sequences by saying that the sequence {a_i} and the sequence {b_i} are "equivalent" if and only if the sequence {a_i−b_i} is a Cauchy sequence.

Note the correction: this should read "the sequence {a_i−b_i} converges to 0".

> It is this latter definition that gives rise to the numerical representation. A decimal expansion
>
> N.d1d2d3....
>
> with N an integer, di an integer between 0 and 9, is short hand for

> > the sequence
> >
> > (N, N+d1/10, N+(d1/10)+(d2/100), ..., N + (d1/10) + (d2/100) + ... + (dn/10^n),...)
> >
> > which can easily be verified is a Cauchy sequence; so the decimal expansion represents the EQUIVALENCE CLASS of cauchy sequences corresponding to this sequence. It is again a trivial exercise to show, for example, that the Cauchy sequence represented by 1.000000... (which is the constant sequence (1,1,1,1,...)) and the Cauchy sequence represented by 0.9999.... (which is the sequence (9/10, 99/100, 999/1000, ...)) are equivalent Cauchy sequence. Therefore, a fortiori, they represent the same "real number".
>
> Tell me what is that trivial exercise.

Can you be bothered to try anything that contradicts your preconceptions or challenges your ignorance, or must everything be done for you?

The first sequence is {a_i} with a_i = 1 for all i. The second sequence is {b_j} with b_j = (10^j−1)/10^j for each j.

By definition, {a_i} is equivalent ot {b_j} if and only if {a_i − b_i} converges to 0. First, let c_i = a_i − b_i. Then

c_i = 1 − [(10^j−1)/10^j] = [10^j − 10^j + 1]/10^j = 1/10^j.

Does the sequence {1/10^j} converge to 0? According to the DEFINITION, the sequence {c_i} converges to 0 if and only if for every N>0 there exists M>0 such that, for all n>M, |c_n − 0| < 1/N.

So, let N>0. Then there exists M such that 10^M > N. Therefore, for all n>M, we have

|c_n−0| = |c_n| = c_n = 1/10^n < 1/10^M < 1/N.

Thus, for every N>0 there exists M>0 such that for all n>M, |c_n−0|<1/N. This proves, BY DEFINITIION, that the sequence {c_i}={a_i−b_i} converges to 0. BY DEFINITION, this means that the sequence {a_i} and the sequence {b_i} are equivalent. This means, BY DEFINITION, that the real number corresponding to the equivalence class of the sequence {a_i} (which was the real number represented by the decimal expansion 1.0000....) and the real number corresponding to the equivalence class of the sequence {b_i} (which was the real number represented by the decimal expansion 0.9999....) are the same real number, since the two equivalence classes are the same.

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"It's not denial. I'm just very selective about
what I accept as reality."
--- Calvin ("Calvin and Hobbes")
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