Re: Question

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Arturo Magidin wrote:

> In article <1146156962.005648.209040@xxxxxxxxxxxxxxxxxxxxxxxxxxx>,
> zuhair <zaljohar@xxxxxxxxx> wrote:
>
> [.snip.]
>
> > > Alternatively, one can define a "distance" of rational
> > > numbers by the
> > > usual means: the distance between a/b and c/d is |ad−bc|/bd,
> > > where
> > > |ad−bc| is the absolute value of ad−bc.
> > >
> > > A sequence of rationals is function from the natural numbers
> > > to the
> > > rationals.
> > >
> > > We say a sequence (a_0, a_1,...) (usually denoted {a_i})
> > > converges to
> > > the rational number Q if and only for every N>0 there exists
> > > M>0 such
> > > that if n>M, then |a_n−Q|<1/N.
> > >
> > > We say a sequence (a_0, a_1, ...) is a "Cauchy sequence" if
> > > and only
> > > if for every N>0 there exists M>0 such that if n,m>M, then
> > > |a_n−a_m|<1/N.
> > >
> > > It is easy to verify that if a sequence converges to some
> > > rational,
> > > then it is Cauchy, though the converse does not hold.
> > >
> > > We can define an equivalence relation among sequences by
> > > saying that
> > > the sequence {a_i} and the sequence {b_i} are "equivalent"
> > > if and only
> > > if the sequence {a_i−b_i} is a Cauchy sequence.

Note the correction: this should read "the sequence $\{a_i - b_i\}$ converges to 0".

> > It is this latter definition that gives rise to the numerical representation. A decimal expansion
> >
> > N.d1d2d3....
> >
> > with N an integer, di an integer between 0 and 9, is short hand for
> > the sequence
> >
> > (N, N+d1/10, N+(d1/10)+(d2/100), ..., N + (d1/10) + (d2/100) + ... + (dn/10^n),...)
> >
> > which can easily be verified is a Cauchy sequence; so the decimal
> > expansion represents the EQUIVALENCE CLASS of cauchy sequences
> > corresponding to this sequence. It is again a trivial exercise to
> > show, for example, that the Cauchy sequence represented by 1.000000... (which is the constant sequence (1,1,1,1,...)) and the
> > Cauchy sequence represented by 0.9999.... (which is the sequence
> > (9/10, 99/100, 999/1000, ...)) are equivalent Cauchy sequence. Therefore, a fortiori, they represent the same "real number".
>
> Tell me what is that trivial exercise.

Can you be bothered to try anything that contradicts your preconceptions or challenges your ignorance, or must everything be done for you?

The first sequence is $\{a_i\}$ with $a_i = 1$ for all i. The second sequence is $\{b_j\}$ with $b_j = (10^j - 1)/10^j$ for each j.

By definition, $\{a_i\}$ is equivalent ot $\{b_j\}$ if and only if $\{a_i - b_i\}$ converges to 0. First, let $c_i = a_i - b_i$. Then

$$c_i = 1 - [(10^j - 1)/10^j] = [10^j - 10^j + 1]/10^j = 1/10^j.$$

Does the sequence $\{1/10^j\}$ converge to 0? According to the DEFINITION, the sequence $\{c_i\}$ converges to 0 if and only if for every $N>0$ there exists $M>0$ such that, for all $n>M$, $|c_n - 0| < 1/N$.

> So, let N>0. Then there exists M such that 10^M > N. Therefore, for all n>M, we have
>
> |c_n−0| = |c_n| = c_n = 1/10^n < 1/10^M < 1/N.
>
> Thus, for every N>0 there exists M>0 such that for all n>M, |c_n−0|<1/N. This proves, BY DEFINITIIION, that the sequence {c_i}={a_i−b_i} converges to 0. BY DEFINITION, this means that the sequence {a_i} and the sequence {b_i} are equivalent. This means, BY DEFINITION, that the real number corresponding to the equivalence class of the sequence {a_i} (which was the real number represented by the decimal expansion 1.0000....) and the real number corresponding to the equivalence class of the sequence {b_i} (which was the real number represented by the decimal expansion 0.9999....) are the same real number, since the two equivalence classes are the same.
>
> −−
> ==
> "It's not denial. I'm just very selective about
> what I accept as reality."
> −−− Calvin ("Calvin and Hobbes")
> ==
>
> Arturo Magidin
> magidin@xxxxxxxxxxxxxxxxx

still there is a problem, when you say converges to zero, at what number of terms in that series this will happen, is it at Omega of c_i or Omega+1 or 2^Omega.

Your Cauchy priniciple didn't mention that. It doesn't differentiate between
0.9999........
which contains Omega of nines in it after the decimal point.

and 0.9999....... 9 which contains Omega+1 of nines in it after the decimal point.

In reality I tend to think that it is at what Cantor called once as "The Absolute Infinity" number of terms in those series that c_i will be zero

or in simple words c_ Absolute infinity = 0

Anything less than that number will have c_i > 0.

Zuhair

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