

Transfinite Ordinal Multiplication

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Hi All

I am reading Introduction to Mathematical Philosophy by Bertrand Russell.

It says that Transfinite Ordinal multiplication is non commutative.

$2 * \Omega$ is different from $\Omega * 2$

so $2+2+2+\dots = \Omega$

While $\Omega + \Omega = 2 * \Omega > \Omega$

I see this a little bit fabricated! or let's say fixed.

Why?

Because I can sum number 2 Ω of times and result in $2 * \Omega$, and on the other hand

I also can sum Ω twice to result in Ω .

How?

To ease visualization of these ordinal summations let us use the unary numeral system.

Zero = Empty row of stars

One = *

Two = **

Three = ***

..

..

..

n = ****...nth*

..

..

..

..

..

Transfinite Ordinal Multiplication

Omega = *****.....

Now Omega + Omega = *****.... + *****.... = *****.... *****.... > *****....

This is clear

While ** + ** + ** + ** + = *****..... = Omega this is also clear.

But it seems as if there is something fishy out their!

I can visually sum ** Omega of times and obtain *****....*****.... and not *****.... see below

* * this is the first double of stars

I will add the next double also wide apart each to the right of the star of the first double as below:

** **

Also the third double can be added in a similar manner to get

*** ***

If I continue for that infinitely the result would be *****....
*****.... = 2 Omega.

From the other hand, I can sum two Omegas to result in one Omega as

below

For simplicity let us denote one of the Omegas as (*) (*) (*).....

Now *****.... + (*) (*) (*)..... = (*) (*) (*) (*)..... = Omega

This last summation can be called Inter-digital summation.

Also If we examine the example of adding the two stars Omega of times to result in

2.Omega we see it is also a form of Inter-digital summation.

So It seems that there are two kinds of multiplication operator.

One is Inter-digital multiplication, and the other is extradigital multiplication.

Of coarse both are non-commutative and each one is the converse of the other.

Transfinite Ordinal Multiplication

Let $(*)$ be intradigital multiplication and $*()$ be extradigital multiplication then in Summary:

$$\Omega (*) 2 = \Omega$$

$$2(*) \Omega = 2.\Omega > \Omega$$

$$\Omega *() 2 = 2.\Omega > \Omega$$

$$2 *() \Omega = \Omega$$

Any Comments?

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