Re: Continuity of complex function question,

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- *From*: José Carlos Santos <jcsantos@xxxxxxxx>
- *Date*: Sun, 30 Apr 2006 21:44:04 +0100

James wrote:

> I am trying to show that
>
> H(z) = int_[0,1] h(t)/(t−z) dt
>
> (where h(t) is a continuous function on [0,1])
>
> is continuous.

It is even analytic.

> I am stuck at one little place :
>
> First of all, h continuous on [0,1] means |h(t)| <= M for some M.

No it doesn't mean that. It just implies it.

> Let |z−z'| < D.
>
> So, | H(z) − H(z') | <= M int_[0,1] (z−z')/[(t−z)(t−z')] <= D*M int_[0,1] 1/[(t−z)(t−z')]
>
> since 1/(t−z) − 1/(t−z') = (z−z')/[(t−z)(t−z')].
>
> But what do I do with int_[0,1] 1/[(t−z)(t−z')]?

Well, you failed to mention what is the domain of H, but I'll assume
that it is C\[0,1]. Let _d_ be the distance from _z_ to [0,1]. Suppose
now that D was chosen such that the distance from each element of the
open disk centered at _z_ with radius D to [0,1] is greater than d/2.
Then, in particular, the distance from z' to [0,1] is greater than d/2.
So, for each _t_ in [0,1], |(t − z)(t − z'| >= d^2/2 and therefore
int_[0,1] |1/(t − z)(t − z') dt <= 2/d^2.

Best regards,

Jose Carlos Santos
in particular,
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