Re: random orthogonal matrix

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On Tue, 25 Apr 2006, Fedor wrote:

> Hello,
>
> for my work, I have to write a program that gives

random orthogonal

> matrices, that is that if mu is the Haar measure on

O_n(R) and S a

> borel set of O_n(R), the probability that my

program gives a matrix in

> S is mu(X). This is why I wonder if the following

algorithm works:

> take (v_1,..,v_n) some random vectors in R^n, each

coordinate of v_k is

> a random number in [−1;+1]. Then, with Gram−Schmidt

one can obtain a

> sequence of unit orthogonal vectors (w_1,..,w_n).

Then the program

> returns [w_1,..,w_n].
>
> Does it work ? If yes, is there a better way to do

this ? (from the

numerical point of view ..)

thanx in advance,
regards

Gaussian, independent, identically distributed
entries are needed. (Why
Gaussian? That is a sufficient condition, as far as I
know − and has a
standard proof − I saw an outline in an older issue
of the Notices of AMS.
The important thing is that the joint distribution is
orthogonally
invariant − just write down the joint probability
density and see).

Caution: the MATLAB routine for QR factorization does
not use
Gram−Schmidt; it uses essentially Householder
reflections. The curious
side effect is that the eigenvalues of the orthogonal
matrices Q thus
generated are not uniformly distributed over the unit
circle − there are
conspicuous gaps near +1 and −1.

For the heck of it, generate a reasonable number of
such matrices (I used
200, 10by10 each) and see for yourself. Can anyone
explain
(quantitatively, if possible) these gaps?

Cheers, ZVK(Slavek).

It is not clear to me that a uniformly random orthogonal matrix (uniform according to the Haar measure) would have eigenvalues that are uniformly distributed on the unit circle in the complex plain. Do you know of a proof that this is so?

I know for certain that a uniformly random member of SO_3 does not have eigenvalues that are uniformly distributed on the unit circle. The eigenvalues are +1, exp(+i*t), exp(−i*t), where i = sqrt(−1) and the distribution of t is not uniform on [0, pi}. The probability density function for t is

d(t) = sin(t)/2

Thus there would be a sparsity of eigenvalues near +1 and −1.

− MO

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