Re: Calculus XOR Probability

Source: http://sci.tech−archive.net/Archive/sci.math/2006−05/msg01027.html

- *From*: Tony Orlow <aeo6@xxxxxxxxxxx>
- *Date*: Fri, 5 May 2006 15:34:30 −0400

cbrown@xxxxxxxxxxxxxxxxx said:

> Tony Orlow wrote:
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>>>>> cbrown@xxxxxxxxxxxxxxxxx
>>>>> said:
>>>
>>> <snip>
>>>
>>>> When you say there's a
>>>> "problem" with my limit,
>>>> what do you mean?
>>>
>>> I mean that it doesn't take into account
>>> anything but location, and you're
>>> using it to measure distance.
>
> Distance and length are real numbers.
>
> Is lim n−>oo {C_n} a real number, or is it a set of points?

It's a sequential set of points, that is a line of some sort, with a real
measure called length.

> No, it's not.
>
> {C_n} is a /sequence/ of /sets of points/.

C_n for n in N is a sequence of sequential sets of elements, which can either be the corners of the steps, or the line segments which connect them.

> A sequence of elements from some set X is essentially a function f : N −> X. We write "f_n" instead of f(n), so we don't get confused and think that f(3/2) might have some meaning. We then write {f_n} to indicate the whole function, rather than its value at some particular n.
>
> (lim n−>oo {C_n}) is defined to be a particular /set of points/. There is no requirement that this limit be "a line of some sort, with a real measure called length".

If that is the type of limit you're using, you have no business claiming it gives a valid measure, then blaming the fact that it doesn't on infinite induction.

> For example, if C_n = {(x,y): x^2 + y^2 <= 1/n + 1}, the limit of this sequence the unit disk: {(x,y): x^2 + y^2 <= 1}. That, to the best I can determine, is not a "line of some sort, with a real measure called length": it is simply a set of points.

Uh, yeah, that's not a line, but a disk. If you replace the "<=" with a "=", then it'll be the circumference, which is a "line of some sort". What you have there is a 2D "space of some sort". You're going to need two directions on your points to measure that. ;)

> These facts can be deduced from the /definition/ of "limit of a sequence of sets of points" that I gave previously.
>
> I don"t know what you mean by "a line of some sort, with a real measure of length", so I can't otherwise comment on whether or not your statement is correct or incorrect; it is meaningless to me, mathematically speaking; although as regular English usage it seems contradicted by the example I just gave.

A disk contradicts a line? Dear god, now you sound like Lester!

> Can you provide a definition of what it means for a set C to be "a
> line of some sort, with a real measure of length"?

E A (endpoints)
E B
E x<>A −> E y: x=successor(y)
E x<>B −> E y: y=successor(x)
x=successor(y) <−> y=predecessor(x)
Length=Sum(x=successor(A)−>B: x−predecessor(x)) where subtraction of points indicates distance, which depends on the dimension of space.

> Can you /then/ show how /my definition/ of lim n−>oo {C_n} satisifies
> /your definition/ of "a line of some sort, with a real measure of
> length"?

Takin the corners of the stairs as your sequence of points, with n stairs, each is 1/n in height and width, and when you sum all those up, you get 2. yes, even if n is infinite. It's the same as the probability problem in that respect, except you've thrown in the conflation of location proximity with identity.

> If you can, then I would accept that the lim n−>oo {C_n} is a "line of
> some sort with a real measure of length".
>
> If you can't then I'm afraid your statement is meaningless to me,
> mathematically speaking. I meant that lim n−>oo {C_n} is a set of
> points; that's why I defined it that way, and not some other way.

Each pair of contiguous points has a distance between them that contributes to the overall length. When the direction at that point is not towards the endpoint, then you are going to get a less than straight line, with length above that given by Pythagorean theorem and the two endpoints.

> > > To measure distance or length of a set of points in R^2 with
> > > the usual
> > > metric, I use the usual approach, just like everyone else.

> > That's good, because this approach doesn't work. When you prove the length

> in
> the limit is 2 instead of sqrt(2), that's an indication that the object in the limit is NOT exactly the same as the diagonal line...

Well, since this point still seems unclear to you, I will amplify on all the definitions used in my argument:

Forget "the diagonal line" for the moment. Forget "figures in the x/y plane". These phrases are unneccesary to my argument, and are confusing you by their usual connotations.

By R I mean the real numbers − not the "real numbers + infinitesimals + infinite numbers".

By R^2, I mean the set of all ordered pairs (a,b) where a and b are real numbers. I will call these "pairs in R^2" or simply "pairs".

By "a set of pairs in R^2" I mean, well, a set of pairs in R^2.

If A and B are two sets of pairs in R^2, then A = B if, and only if, for every pair p, p in A if and only if p in B.

If A = B, and f is any function that takes a set of pairs in R^2 to a real number, then f(A) = f(B). Thus if A = B, A is identical to B "in every way" except by name.

By "a metric on R^2", I mean that I define a function dist(p,q) for all pairs p = (p_a, p_b) and q = (q_a, q_b) in R^2 by:

dist(p,q) = sqrt((p_a−q_a)^2 + (p_b−q_b)^2).

Thus dist is a function from two pairs in R^2, that yields a real number.

I observe the following, which you are free to prove me wrong about:

* dist(p,q) = 0 if, and only if, p = q.
* dist(p,q) = dist(q,p) for all pairs p, q.
* For all pairs p,q, and r, dist(p,q)+dist(q,r) >= dist(p,r).

Thus, the function "dist" is a (toplogical) metric by the usual definition: it satisfies the three conditions above. That makes R^2, with reference to the function dist, a "metric space" as defined by the metric "dist".

(One could define other functions which would also be a metric on R^2. By "the usual metric" in R^n for any n, I mean that the dist function we want to use is the same as euclidean distance in R^n between the two points p and q).

Okay, but what happens in the limit, in your estimation? When the distances between successive points become less than any real number, you seem to think that the distance is zero or something. But then, how do you account for the length of the line? How do those zero lengths sum to the overall length of the line? If you simply want to forget about infinitesimal line segments, then you really can't handle this object. This works well for measuring the diagonal, but not for summing infinitesimal segments, such as the staircase in the limit, without some notion of infinitesmals.

> −−−−−−−−−−−−−−−−−−−−−−
>
> What do I mean by "C is a curve"?
>
> Suppose C is a set of pairs in R^2. Call such a set a "curve" if there is a function f: R −> R^2 such that the point
>
> (1) the pair f(x) is in C for all x in R;
>
> (2) for all pairs p in C, there exists a unique x in R such that p = f(x); which I will write as f^−1(p).
>
> (2) if e is a real number greater than 0, and p is a point in C, then there exists a real number d such that if q is a pair in C, and dist(p, q) < e, then |f^−1(p) − f^−1(q)| < d.
>
> (This is what is I mean by "there exists a continuous function f from the real line to C".)

Okay. You're defining a linear continuum as a curve. Basically, you're mapping the real line to any curve. That's fine. Except, how do you determine which r in R any given point corresponds to, besides the first?

> −−−−−−−−−−−−−−−−−−−−−−−
>
> Let C be the set of pairs defined by {p = (a,b) : b = 1 − a}. Is C a "curve"? Well, let f be the function defined as f(x) = (x, 1−x).
>
> (1) Is f(x) in C for all x in R?

If one uses the x coordinate as the real x in R, does that mean your curve must have infinite domain? No, that's jus one way to do it, but okay...

> (2) If p is a point in C, is there a unique x in R with f(x) = p?
>
> (3) if e is a real number greater than 0, and p is a point in C, is there a real number d such that for all q in C with dist(p,q) < e, it is always the case that |f^−1(p) − f^−1(q)| < d?
>
> The answer to each of the above is "yes" (feel free to prove me wrong); so C is a "curve", because the function f exists and satisfies the three rules I gave in the definition of "C is a curve".

Okay.

> −−−−−−−−−−−−−−−−−−−−−−−−−−
>
> Now suppose we define a "segment" C_[u,v] of a "curve" C as the subset of C defined by {p : p in C and u <= f^−1(p) <= v}.

Alright, as long as we can determine where f(u) and f(v) lie.

> Let C be the set of pairs defined by {p = (a,b) : b = 1 − a}; and let D = C_[0,1]; then D = {p = (a,b) : b = 1−a, a,b>=0} (feel free to prove me wrong).

No, I can live with that.

> −−−−−−−−−−−−−−−−−−−−
>
> If D = C_[u,v] is a segment of a "curve" C, then define the "length" of D as follows:
>
> (1) Call a finite sequence of n reals {a_0, a_1, ..., a_(n−1)} a "joint set" if a_0 = u, a_(n−1) = v, and a_i < a_j for all i, j. So a "joint set" is basically a finite set of points in [0,1] that includes 0 and 1; and where the points in the sequence are "in order". For example if [u,v] = [0,1], (0, 1/2, 3/4, 7/8, 1) is a joint set, but (0, 3/4, 1/2, 7/8, 1)
>
> (2) Recall that because weare saying that C is a curve, there is a continuous function f : [0,1] −> D. Define the "joint length" of a joint set J = {a_0, a_1,..., a_(n−1)} as the sum

dist(f(a_0), f(a_1))
+ dist(f(a_1), f(a_2)) + ...
+ dist(f(a_(n−2), f(a_(n−1)).

(3) Define the "length" of D as the /smallest/ real number which is /greater than/ or equal to the "joint length" of any "joint set".

Okay, but that's only for a finite sequence so far.....

−−−−−−−−−−−−−−−−−−−−−−

Do you now understand what I mean by the "length" of the set D of pairs in R^2? It's simply a real number, that I can define for a segment D of any set of pairs C that also satisfies the definition of "curve".

Can you see that by this definition, the "length" of the segment D = {(a,b) : b = 1 − a, a,b >= 0}, which is simply a set of pairs, is the real number sqrt(2)? If not, the proof is not difficult.

Can you see that by this definition the "length" of the nth staircase is always 2, regardless of n? Again, the proof is not difficult; but it is tedious.

That is what I mean by "the usual method of measuring the length of a curve". It is a well−defined method of turning any set of pairs in R^2 which meets certain requirements, into a real number; which we then call the "length" of that curve.

Well, thank you for that lengthy explanation. It makes sense. It's just that I find myself wondering, if this rigorous definition of the length of the curve gives the result that the length of the staircase in the limit is 2, then why do you question that result, and not the exact equivalence between that curve and the diagonal? I think that result is ultimately correct.

<snip>

Do you understand what I'm saying about the elements of the staircase needing
to be parallel to the diagonal in order for the "usual metric" to work?

No, not really; but that is irrelevant to my argument.

I'm not talking about "needing to be parallel" in order for "the usual metric to work". I'm talking about two different well−defined functions:

(1) A limit function: whose domain is /any/ sequence {C_n} of sets of pairs in R^2, which allows us to unambiguously calculate a unique set of pairs C, which we write as "C = lim n−>oo {C_n}".

(2) A length function: as described above, whose domain is sets of pairs C which obey the definition "segment of a curve" given above, and which yields a unique real number which we call the "length" of the set of pairs C.

My claim is that, under these definitions, it is not always the case that length(lim n−>oo {C_n}) = lim n−>oo {length(C_n)}.

That depends, as you yourself stated, on how you define your limit. No?

In particular, I can explicitly provide a sequence {C_n} where length(C_n) = 2 for all n, thus lim n−>oo {length(C_n)} = 2; but length(lim n−>oo {C_n}) = sqrt(2).

That's only because you equate the two objects which have obviously different measures.

That would "break" your "infinite induction" principle, would it not?

No, for the same reason I have reiterated. They are two different objects. The error isn't in infinite induction, but the failure to make this distinction.

<snip>

No, you are misunderstanding me. I reject your example as demonstrating that
the concept of an inductive proof of equality holding in the infinite case is invalid, because the error in your staircase example is easily explainable in the way I've been describing.

Your description of "the error" appears to be based on a misconception: you think that the length of a curve can only be unambigously defined when the definition of "curve" is something other than "a set of points

> in R^2 with some additional restrictions".
>
> But I /have/ given above a definition of "length" which is unambiguously defined for sets of pairs in R^2 meeting certain restrictions.

And guess what? The length is 2. Huh!

> Can you make the mental leap betwen "sets of pairs in R^2" and "sets of points in R^2"? Just substitute the word "point" for "pair", and "euclidean distance" for "dist". That's what "R^2 with the usual metric" means.

Yes, I know, a pair of Cartesian coordinates.

> You may complain that then my example isn't "really" figures in the x/y plane anymore.

No, I understand this is on the Cartesian plane, and therefore each point is a pair of real coordinates.

> Fine; since it is irrelevant to providing you with a counter−example to "infinite induction", continue to call them "sets of pairs in R^2".
>
> (1) Do you think I can't define a sequence of staircases {C_n} as a sequence of sets of pairs in R^2? I have already done this for you at least once: see, for example, the end of:
>
> http://groups.google.com/group/sci.math/msg/8b80a8687cead0c6?&hl=en&q=+risers+treads

That seems to point to this message. :(

> (2) Do you think that the limit of that sequence is not the set of pairs D = {(a,b) : b = 1−a, a,b <= 0}? Again this limit is proven at the same link as above.
>
> (3) Do you think that the "length" of the set D is other than sqrt(2)?
>
> (Sketch of proof: select n points in the interval [0,1] including 0 and

> 1. Order them as them as the sequence {a_0, a_1, ..., a_(n−1)}. Let f(x) = (x, 1−x). Calculate the sum i = 1 to n−1 dist(f(a_(i−1), f(i)). Conclude that this sum is constant independent of n, and equals sqrt(2). Therefore, since sqrt(2) is the smallest real number >= sqrt(2), the length of D is sqrt(2).)

When you define the points using y=1−x, then this works.

> (4) Do you think that it is not the case that for any of the staircases in {C_n}, the "length" of that set is other than 2? Do you require another tedious proof of this rather obvious fact as well?
>
> (Sketch of proof: show that the "length" of a segment of a curve is the sum of the lengths of any finite disjoint union of smaller segments of the segment of the curve. Show that any tread or riser of the nth staircase has "length" 1/n, via argument similar to the one given above. The staircase is the disjoint union of the n treads and n risers. Thus the length is n*(1/n + 1/n) = 2).

Yes, the length reamins at 2, even in the limit.

> (5) Do you think that the "length" of lim n−>oo {C_n} = lim n−>oo (the "length" of the set C_n); in other words, do you think that sqrt(2) = 2?

No, I think lim(staircase)<>diagonal, using any limit suitable for measuring length.

> (6) Doesn't that contradict the principle of "infinite induction"?

No, it contradicts the notion that the diagonal is an infinitesimal staircase.

> <snip>
>
> > What is the usual method for measuring the staircase in the limit? As far as I'm concerned, you proved it had length 2.

> That is because you are not using the same definition of "limit", "length", and "curve" that I am using (and you are also not following the proof).

According to your definitions, the length is 2, as long as you don't claim it's the diagonal.

> To that end I gave a full definittion of "length" above; it applies to sets of pairs in R^2 which are also curves, not "sets of points only if they have directions" or "a sequence in the limit", unless by those things you mean exactly and only "a set of pairs in R^2 which is a segment of a curve".

Didn't you prove that way that the length is also 2?

> If the limit of the staircases is the set D of pairs in R^2, there is no difference between "measuring (the length of) the staircase in the limit", and "measuring (the length of) of the set of pairs D"; they are /defined/ to be the same thing.

No, one is defined to be a pointwise limit of the other.

> The "length" of a set D of pairs in R^2 is always the same, no matter how you "get" that set of pairs, whether by formula or some other means such as the limit of a sequence.
>
> That definition is what I meant by "the usual method".

And it seems to support your original proof, does it not?

> > > Finally, I use the principle of "infinite induction" to /deduce/ that
> > > "length(limit)=limit(length)"; i.e.:

Yes, that's very nice, and I have no problem with that. You're measuring the length of an infinite number of infinitesimal steps, not a straight line.

I'm measuring the "length" of a set of pairs in R^2. That set of pairs D is always the same thing; it isn't "sometimes a line, and at other times a fractal line": it is simply a set of pairs in R^2. Two sets of pairs are equal if and only if each contains the same set of pairs. Period.

If and only if equal. Equality is a funny thing. It depends on distinguishability, and that depends on the method of measure.

... the principle of infinite induction claims that I /can always/
"use" the limit of the lengths of the curves "to measure" the length of
the limit of the curves.

Yeah, and that worked out pretty good, didn't it?

If you consider "2 = sqrt(2)" working out "pretty good", I suppose. I consider it a contradiction, that implies that your principle of induction doesn't hold in every case.

No, it implies that the diagonal is different from the fractal diagonal.

It is this /third/ assertion (premise B, "infinite induction") which
causes a "problem", not the first two.

No, you're just confused because you think that location is all there is to a set of points.

You're confused because you think it is sensible, when someone is

> making an argument about sets of points in R^2, to insert comments
> about some other thing: "points with directions". This is what I mean
> by "2*x = 3 is satisfied by the natural number 3/2".

If I want to talk about splitting 2 pies between 3 people, and want to restrict the conversation to whole numbers, that's a pretty hopeless coversation, isn't it? If you want to bring arc length into the conversation, but don't want to talk about the methods that make approximating arc length work, then what's the point?

> Sets of points with directions, whatever they might be, are /different
> things/ than sets of points, just as rational numbers are /different
> things/ than natural numbers. My argument is regarding the latter, not
> the former; and I /explicitly stated/ that.

Yes, and then you went on to make conclusions about the length of the staircase on the assumption that your set of points was sufficient to measure that. If you want to split your pies like that, then you better get a knife. In other words, don't talk about limits of curves that don't provide a good measure, and then blame that on something else. You wanna talk about measure? Points don't cut it. You need the lines between points to get measure.

> > When you're talking about a curve, you're talking about an
> > uncountable sequence of points, and there is a connectedness between
> > successive
> > points that has direction.
>
> No, that's not what I'm talking about, and that's why your objections
> are "meaningless" to me, mathematically speaking.
>
> When I'm talking about a curve, I'm talking about a /set/ of /points/
> in /R^2/; such that there exists a continuous mapping to the real line,
> under the metric topology induced by the euclidean distance between
> points. And that's all I mean; nothing else.

Not much difference ultimately.

> I don't know /what/ you're talking about when you're talking about a
> curve; but it really doesn't matter. Because my argument is not about
> what /you mean/ by a curve; it's about what /I have defined a curve to

mean for the purposes of this argument/.

It is one thing to fail to understand what /I/ mean by "a curve", and consider my argument meaningless. You can always ask for clarifcation of any terms I use; and I gave ample clarifications above. If there are still terms you find unclear, feel free to ask for clarification.

But it is entirely another thing to confuse my argument with someone else's /different/ argument, because you fail to apply the definitions I give in my argument, and/or substitute your own definitions instead.

Until you learn this distinction, you will never be able to make a mathematcial argument yourself.

Okay, so your limit is exactly the diagonal, even though your rigorous definition of arc length gives a different value than for the diagonal. I can see we'll never agree on this. These kinds of "counterexamples" are all over the place, "disproving" all sorts of things on the basis of something else.

> > One can just as easily view a curve as a set of infinitesimal segments between points, as a set of points separated by infinitesimal segments.

> One can just as easily consider it anything at all one likes: a road that isn't straight, the shape of a woman's hip, a kind of baseball pitch, or a particularly obscure turn of phrase.
>
> And that would be then be a /different argument/. This is /this argument/; so such flights of fancy are no more relevant than stating "we can view 3/2 is a natural number".

> > > The third assertion is simply false: limit(length of curve) is not
> > > equal to length(limit of curves), where "limit", "length", and "curve"
> > > refer to objects in the domain of R^2 with the usual metric. Therefore,
> > > the principle of "infinite induction" /does not apply/ in the case
> > > where we are working in R^2 with the usual metric; and I think you'll
> > > find that it doesn't apply in most situations.

> > What do you mean by "the usual metric". Maybe this is the problem here. Is

the
metric usually wrong?

I gave you a definition of metric above; hopefully this answers your question. "The usual metric" in R^n is the function yielding the euclidean distance between two points.

I have no idea what you mean by a metric being "wrong". It's simply a function, obeying some properties.

No, the usual metric isn't wrong, but you're not really using it on the staircase, when you ignore the infinitesimal right angles in it. The distances between elements approach 0, and at that point you think the corners disappear, but they don't.

There is no reason to tack on additional ill−defined (or even well−defined) concepts like "therefore there are points with no, some,
and all directions, at infinitesimally different locations" that are
not part of my argument, and are certainly not a part of R^2 with the
usual metric, which is, after all, the usual domain of curves, including all the familar the results of calculus.

Yeah, well, as I understand it, the usual metric is generally used parallel to whatever one is measuring.

Well, "as you understand it" is not what I mean why I say "the usual metric". So I gave you a definition of "the usual metric". Do you now understand what I mean by "the usual metric"?

Sure, and as you stated, d(x,z)<=d(x,y)+d(y,z). In this case, d(x,z)=(d(x,y)+d(y,z))/sqrt(2).

It's function, that is also a (toplogical) metric: the usual metric is the function which gives the euclidean distance between two points. You could use other metrics if you like; some of the more bizzare ones are quite interesting. That would be of course a /different/ argument than the one I am giving.

> > To add these other "features" of points in is an amusing
> > diversion; but
> > it is a diversion: it is like saying "the natural number
> > satisfying
> > '2*x = 3' is x = 3/2". Fine, but 3/2 is not a natural number.

> No, it's nothing like that.

Yes, it is /exactly/ like that.

When you say "the usual metric is generally used parallel to whatever one is measuring", you are saying "3/2 is a natural". A metric is not something that you "use" "parallel" to something that you are "measuring".

I am saying that approximating a curve is done parallel to the curve, or done wrong.

It is simply a function d over a set X of the form d: X^2 −> R that yields a real number greater than or equal to 0, which obeys some additional rules for all p,q, and r in X:

* d(p,q) = 0 if, and only if, p = q.
* d(p,q) = d(q,p)
* For all p,q, and r, d(p,q)+d(q,r) >= d(p,r).

Thus
"2*x = 3 for some natural"
is to
"x = 3/2"
as
"the length of a set of points can be determined via a metric"
is to
"but a metric is generally used parallel to whatever one is
measuring"

Similarly, when you claim that the length of D = {(a,b): b = 1− a, a,b>= 0} is anything other than 2, you are not refering to my argument's definition of length which is defined on /sets/ of /points/; you are refering to /some other/ definition of "length" that is used on /some other/ kind of "figure in the x/y plane" (which apparently consists of "sets of points with directions" and other notions).

Thus
"2*x = 3 for some natural x"
is to
"x = 3/2"
as
"the length of the set D of points in R^2 is sqrt(2)"
is to
"no; it must be 2, because D is actually a set of points with
directions."

> As far as I'm concerned, the error of sqrt(2) is the inverse of the
> cosine of the 45 degree angle each riser and tread makes with the diagonal.
> The
> accurate measure of curves depends directly on parallel metrics.

How do statements like this address any of the logical steps in my
argument? How does "inverse cosine" apply to a /set/ of /pairs/ in R^2?

Given p1, p2 and p3 in C, the distance between p1 and p3 will be the sum of d
(p1,p2) and d(p2,p3) times the cosine of the angle between the segments p1p2
and p2p3. So, if the individual segments are summed, you will get an answer
that is the distance between the endpoints, divided by this cosine.

> > This is not true only in R^2 with the usual metric; in fact,
> > you have
> > to work very carefully for your "infinite induction" to hold in
> > a
> > paricular domain.
> >
> > I don't think you (or Han) are being that careful, in general,
> > when you
> > invoke it; and that is the point I have been trying to make.
>
> Well, I think you're wrong, and I don't think you're example illuminated
> anything other than basic calculus ideas of approximation.

It's increasingly hard to imagine how you expect any mathematical
argument to convince you of something about which you have already
decided "it is simply true".

> Suppose I have a set X.
>
> Suppose I define a limit on sequences of X such that lim n−>oo {x_n} is well−defined for all sequences of elements in X.
>
> Suppose I define a function f : X −> R (the real numbers).
>
> Now, suppose as a /result/ of these /definitions/, I can explicitly provide a sequence {x_n}, such that I can provide a proof that f(x_n) = 1 for all n, but I can also prove that f(lim n−>oo {x_n}) = 0.
>
> I'm not claiming at the moment I have neccesarily done this; I'm just asking you to /suppose/ that I had, for the sake of argument.

I would then conclude that whatever notion of limit you are using is inadequate for measuring whatever f(x_n) is supposed to be measuring.

> Which, if any, of the following would you then claim?
>
> (1) Therefore, "infinite induction" does not hold in this case.

Not when there is another obvious explanation for the error.

> (2) Therefore, lim n−>oo {x_n} is actually not a member of X, it is something else.

No, I wouldn't say that.

> (3) Therefore, f(x_n) is actually not 1 for some natural n.

I would say, given that it's inductively provable that it is, then it is.

> (4) Therefore, lim n−>oo {x_n} is not "the infinite case".

I would say there is an issue between your limit and f(x).

> (5) {Insert other possible objection here}.
>
> You are currently claiming "(2)", in our non−hypothetical example of the staircases.

Maybe I read that wrong. I would say that there is a discrepancy between your notion of limit and the measure you are evaluating.

> Suppose I can convince you that (2) is false (i.e., by restricting our discussion solely to "sets of pairs in R^2").
>
> If (4) is what you would then claim, then what /is/ "the infinite case" in your principle of "infinite induction"?

When you are proving an equality between real expressions, then it holds in all cases, and nothing "magical" happens at oo.

> > It has nothing to do
> > with probability over an infinite set, and didn't show why infinite induction of equalities ever fails. It's just your assumption of "usual metric" that fails.
>
> That "fails" /how/?
>
> * Fails to be a function of the form (R^2) x (R^2) −> R?
> * Fails to be a (topological) metric?
> * Fails to provide an unambiguous definition of "length" for any set of pairs in R^2 which is also a "curve" by my definition?
> * Fails to unambiguously define whether or not any pair p is a limit point of some sequence {C_n} of sets of pairs in R^2?
> * Fails to unambiguously define a set of pairs in R^2 which is the limit of any sequence {C_n} of sets of pairs in R^2?
> * Fails in /any way/ to obey the /premises/ of your assertion?
>
> Or, is it that it simply fails to support your assertion that "infinite induction" is always applicable, while still satisfying your premises? In that case, why would you continue to assert your principle? It would be false.

It's not the "usual metric" that fails. I misspoke. It's the false equivalence between the diagonal and the staircase in the limit.

<snip>

So it is simultaneously directionless, and yet parallel to two different lines, which are at right angles to each other?

Use your mind's eye. Each side is parallel to one of the axes.

This isn't Poetry. It's Mathematics. You don't "use your minds eye" to define terms; you use precise, unambiguous mathematical definitions. That's part of what makes mathematics a different thing than poetry.

So, you also didn;'t understand what a square parallel to the axes might mean? What else could it possibly mean?

According to the definitions I gave, each of the above formulas
identically yields the single point (0,0) in R^2 with the usual metric.
In other words, the figure in the x/y plane described by "{(x,y) :
f(x,y)=c}" for all the various functions f is identical: they are not
merely "indistinguishable", they are equal.

Single points don't have any metric anyway.

Since it is clear that at this point, you didn't know what a "metric" is, your statement is meaningless. Hopefully now you can see that if X = {a} is a singleton set; the only possible metric on X would be f(a,a) = 0.

Uh, right, a measure of zero.

You claim that, essentially, they are different: but you are using
logic equivalent to "but there /is/ a natural number such that 2*x =3;
it is x = 3/2".

No, that is about as related to what I am saying as your example is to the original topic. Like, not.

In your post of April 26,

http://groups.google.com/group/sci.math/msg/364a1721e9c9c13f?dmode=source

you said:

"Now, I would pose this suggestion again. Inductive proof is considered generally invalid "in the limit", that is, in the infinite case. The method of inductive proof is only considered to prove a property for all FINITE n. However, in this sense of a limit, my suggestion is that an equality between expressions proven inductively holds for all n, finite or infinite. I don't think that causes any problems such as what you've been suggesting, do you? "

My example of the staircases is related to thetopic, in that I am attempting to answer the question you posed by providing you with a "believable counte−rexample" to the principle of "infinite induction" espoused above that causes the same sorts of "problems" as I suggested.

Perhaps you could present an example that isn't confounded by metrics and other issues. I doubt it.

This is hampered by your inexperience at understanding what is meant /mathematically/ by a "believable counter−example"; i.e., a /proof/ that a counter−example exists which satisfies your premises, and yet contradicts your assertion.

It was never my premise that the staircase in the limit is exactly the diagonal in every sense.

This in turn exposes your inexperience at understanding what is meant by a proof in general; and in particular, what is meant by a mathematical definition.

Thus these long and doubtless frustrating explanations. Ah well; perhaps you'll pick it up as you go along.

<snip>

> I said that in response to the circle in the limit. What direction does a circle have? (groan)

/I/ don't claim the circle has "a direction" at all; that phrase is meaningless to me. You seem to claim that it does or at least may "a direction"; and that's why I asked for a /definition/ of the term. I'm not a mind reader; so as you have given no definition that I can apply to, for example, "the direction of an ellipse", aside from phrases like "just use your mind's eye", the only way for me to determine it currently is to say "ask Tony; that phrase is meaningless to me, mathematically speaking".

<snip>

> Look, in the standard sense, what you say is correct about the distinguishability of those points. So what?

So what!?!

So therefore, my argument shows that "infinite induction" doesn't hold in every sense; in particular, it doesn't hold in the "standard sense" of R^2 with the usual metric (without "points with directions", however you may choose to define them). That's what you /asked me/ to /provide/.

My argument is based on: R^2, the usual metric, the standard definitions of limit, length, and curve; and the /logical deductions/ which follow from these /definitions/.

One logical deduction is "length(limit of curves) is not equal to limit of(length of nth curve)". And thus, your assertion of "infinite induction" fails to hold in the "standard" case.

The fact that my argument /doesn't apply/ in some /other case/ is irrelevant to the argument as a disproof of the assertion: "the principle of infinite induction holds in every case".

It disproves it by providing a counter−example under particular assumptions: R, R^2, euclidean distance, sets of points, limits.

> Therefore, the conclusion is: There exist conditions (assumptions and definitions) under which your principle doesn't apply.

Or, there exist definitions of "limit" which do not preserve measure.

> Therefore, you cannot simply assert that your principle is true, without also /proving/ that it is not false under the /particular/ assumptions and definitions /you/ are considering: e.g., "R + infinitesimals + other ad hoc additions", "infinite probability distributions", etc.: /regardless/ of how you choose to define those terms, and whose definitions I'm not particularly interested in.
>
> Because my example shows that it is /at least possible/ that under your definitions, your principle will /also/ not hold.

It's either infinite induction, or pointwise limits with proper measure. One of those two concepts is at work here.

> > You're "usual metric" didn't work
> > for precisely the reasons I've explained, and you can't blame it on infinite induction.

> Your critique of my proof has so far hinged on your assertion that there are two different objects under consideration: the "real" diagonal line, and the "thing" which is the limit of the staircases, which may /look like/ the diagonal line,and indeed be "indistinguishable from" the diagonal line, but /isn't/ the diagonal line (as well as numerous other misunderstandings of "metric" and so on)
>
> Hopefully, by "figures in the x/y plane", you can now see that I am talking solely about "pairs in R^2, using the function dist() as a metric". And that by "limit", "curve" and "length", I mean very specific and limited statements about sets of pairs in R^2 and sequences of same.

Yes, points are pairs, and disatnces are pairs of pairs.You have your pairs in your limit, but not your pairs of pairs which are required by the dist() metric. It's those pairs of pairs which are distinguishable from those in the diagonal by always having a corresponding element equal from one pair to the other.

> Thus there is exactly one set I mean by the set of pairs D = {(a,b) : b = 1− a, a,b >= 0}. It has "length" sqrt(2). It is the same set (by set membership, which is all one means by "identical" as regards sets) as the limit of the staircases (redefined as sets of pairsin R^2). And yet the nth staircase, which is a set of pairs in R^2, always has "length" 2, regardless of n.
>
> So under those assumptions, "infinite induction" fails: sqrt(2) = length(D) = length(limit {C_n})) <> limit {length(C_n)} = 2.
>
> Therefore, "infinite induction" doesn't always hold. And that is what you /asked me/ to prove.
>
> Therefore you must /prove/ that your principle /does/ hold when you use it in any other argument.

No, you must disprove it with a pure example. Yours had another explanation. I maintain that it holds.

> If, that is, what you want to do is make a mathematical argument. Otherwise, you can say whatever you like; but it would be off−topic in sci.math, except for entertainment's sake. Expect to then be mocked and maligned by some posters, at the minimum: this is, after all, usenet.

Yeah, like that hasn't happened once or twice.

> <snip>
>
>> No, that's far more complicated than it has to be. A point in space need not have any direction. A point on a curve has direction as long as it doesn't have a derivative discontinuity, that is, a corner. A curve is a sequence of points, or of segments between points with direction.
>
> How is what you just described relevant to "sets of pairs in R^2"? If it isn't relevant to that, then what you are talking about is not what my argument is about.

If it's about a metric, the it's about distance between those pairs, not the pairs in isolation.

My argument is about sets of pairs in R^2; and well−defined functions on those sets such as "length" and "limit". "Derivative continuity" and "sequence of points" are not relevant to the discussion, unless you can express those terms as unambiguous statements regarding sets of pairs in R^2, and the definitions of "length", "curve" and "limit" that I use in the argument.

<snip>

Thus

"is there a natural number such that 2*x =3?"

is to

"yes, 3/2"

as

"is there a difference in R^2 with the usual metric between the
figures x^2 + y^2 = 0 and lim n−>oo {y=x,−1/n< x< 1/n}?"

is to

"yes; they have different directions because they are different elements of X^2 x P(X union {"none"})\{}".

Straw man. That's not my argument.

It is indeed your argument: you argue by use of undefined terms which are not relevant to the argument, or redefine terms that are defined in the argument to mean something else (or both simultaneously, as is usually the case).

Unless you can rephrase your statement "A curve is a sequence of points, or of segments between points with direction" as a statement limited exclusively to sets of pairs in R^2, the definition of such a set being a "curve", the defined "length" function on those sets which are "curves", the defined "limit" function on sequences of those sets, and the usual mathematical operations on R and rules of inference, then you are /not talking about/ sets of pairs in R^2; you are talking about /some other thing/ than what my argument is talking about. 3/2 is a not a natural number.

> It's not my job to "guess" what you mean by a statement like "Directions as tangent lines are elements of curves that exist conceptually at each point on the curve." (Although I admit it is entertaining as a sidebar to try to formalize such vague statements into actual meaningful mathematical statements.)
>
> Such a statement doesn't stand as a refutation of my argument, until you can translate it into a statement about pairs in R^2, the defined limit function, the defined length function, the definition of curve, and the usual mathematical operations on R.
>
> /Then/ you can say "therefore since {whatever you end up with}, we see that there is a contradiction between {some statement in my arguiment about the set of pairs in R^2 defined by D} and {some other statement of my argument}.
>
> Until then, you are talking about something else; such as 3/2 in the context of the natural numbers. And as such, your statements are meaningless, mathematically speaking.

Fine, I don't have time right now to formulate this for you. I'll think about it and respond later, but my immediate thought is that the pairs that define the points in the curve can be replaced by a sequence of pairs that denote the x and y changes of a series of segments, which in the limit approach the curve. In this sense, the pairs that define the staircase, one of the elements of which is always 0, do not approach the pairs that define the diagonal, which always have x=−y.

> <snip>
>
>> I think we're just going to have to agree to disagree. The problem with your example is obvious. I'm am not convinced.
>
> The /problem/ is that you don't know what is meant by "a mathematical argument", i.e., a proof.
>
> If you did, you would have seen weeks ago that it is obvious that your principle of infinite induction is contradicted by my argument: which is in fact a proof that there exist a counter−example to your assertion that satisfies its premises, as you requested.

Then you're being clueless. You should be able to parse that statement. Or, did you read "probabilities" as "possibilities" again? Remedial English is down the hall to the left.

This is Remedial Mathematics. The current topic is "what is a valid mathematical argument?". Stay in your seat until the bell rings.

If that's your attitude, the discussion's over. Have a nice day.

Ah well. Sorry you feel that way. See ya round then!

I'm not going to tolerate that kind of condescending nonsense.

But I should tolerate being told to that I am "clueless", and that I should "go to Remedial English"? You don't consider that a condescending or otherwise insulting remark, which is not to the point of the argument?

It seems to me if you are going to dish out "jovial" insults of this sort, you should be willing to accept them as well.

I believe I have
a valid point, and that your refutation by counterexample was invalid. We've strayed far from the topic, and now you want to make this a remedial class?

Sorry, the remedy isn't needed, thanks anyway.

It is not being condescending for me to observe that your knowledge of what makes a valid mathematical argument is sorely lacking; anymore than to observe that if you ignored red lights and stop signs and then parked your car on my lawn, I would consider your ability to drive properly to be sorely lacking.

Nor is it condescending for me to observe that I know a lot more about mathematical arguments than you do. Which is not to say that aren't other people who know a lot more about mathematical arguments than I do.

You may find that my judgement "stings" or is "unfair", and I'm sorry if you feel that way; particularly since it seems to impede your ability to learn. But it's my honest judgement, and I feel very confident in these observations; observations that appear to be shared by most other observers.

If you're interested in mathematics, then you need to /learn/ some mathematics; because what you are doing now is not mathematics, it is some other thing.

Since you are posting in sci.math, I assume you are interested in learning mathematics. If you aren't, why /are/ you posting here?

Cheers − Chas

−−
Smiles,

Tony
.