

# Re: Calculus XOR Probability

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- *From:* Tony Orlow <[aeo6@xxxxxxxxxxxx](mailto:aeo6@xxxxxxxxxxxx)>
  - *Date:* Mon, 15 May 2006 15:51:34 -0400
- 

Mike Kelly said:

Tony Orlow wrote:

<snips>

To say  
all elements are finite but the set is infinite implies that there is some  
largest finite with an infinite successor.

No, it doesn't. It implies there are an infinite (UNBOUNDED) number of  
finites, each with one successor.

Unboundedly large but finite number. None is infinitely after any other. They  
are all within a finite number of steps from one another. But you can continue  
repeating whatever's in your book if you want.

So, it's nothing so simple as "infinite sets all  
have infinite values". The  
reals, or rationals, in  $[0,1]$  are a trivial  
counterexample. This has  
specifically to do with the naturals, and  
generally to do with sets that are  
everywhere sparse. Such sets require infinite  
value range to contain an  
infinite number of elements.

So the (standard) naturals have a finite value range? What is  
it?

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The value of the largest of them minus the smallest, but that's not a specific number in the finite naturals, is it? However, since all values are finite, all differences between values are finite, and there is no  $x-y$  that is infinite. So, while the value range is unboundedly large, it cannot be infinite in this sense.

What is "a specific number"?

A point on the line, on the complex plane, or whatever quantitative space you have defined for your system.

I hope that cleared up the mystery of my position.

It highlighted a couple of areas where you just plain don't seem to understand the standard theory. It didn't explain your position much, it attacked the standard definitions.

Haha. I understand the standard theory. I disagree with some core concepts. If it makes you feel better to say I just don't understand, go ahead, but it's not the case.

It doesn't make me feel better. Maybe when you realise that the dozens of mathematicians saying that you don't understand standard theory are right you might learn something about it?

Yes, and maybe when the hoards of Jehovah's witnesses descend upon your door you'll convert. They're very happy people, I hear.

But I don't think there is a countable sequence of infinite elements that starts at, say 0 and steps through an infinite number of reals in  $[0,1]$  and then ends with 1. I

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think sequences either have  
a start and  
an end in which case they  
have a finite length or they  
lack an end and  
then are infinite sequences.

That's the standard treatment. It appears that the concept of an uncountable sequence with beginning and end is foreign to most people, but if you define infinitesimal units, you can certainly conceive of such a sequence in  $[0,1]$ . When I look at a bit string of all 0's with a 1 in the  $x$  position, I don't care whether  $x$  is finite or infinite. That number is  $2^x$ . I don't need to be able to count up to that bit from the 0 bit to calculate that. So, that also constitutes a completed infinity with distinct ends.

No, I really can't conceive of how this makes any sense to you. What does " $2^x$ " mean to you when  $x$  is not a real number? How do you evaluate it?

First of all I evaluate it compared to other values finitely close in representation, such as  $2^{(x+1)}=2*2^x$ . When it comes to comparing it with values indicated by bits infinitely far away, one must choose some unit infinity, and express that relationship formulaically using that value as a variable.

Gibberish.

You mean, you don't understand what I said, and you'd like some clarification, right?

Consider  
....0001000.... and  
....0010000....

We don't know where the digital point is on either one. It could be infinitely to the right or left, for all we care. If we know the digital point is in the

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same place in both, then we can say without any reservations that the second is twice the value of the first.

Consider that we continue to move our bit to the left. Each time we move one bit to the left, we double the number. If we move  $x$  bits, we have multiplied the original value by  $2^x$ . If we move some particular  $oo$  to the left, we have the original value times  $2^{oo}$ . That's a formulaic relationship.

Perhaps you can be more specific about your question next time.

Yes,  
it  
is  
impossible  
for  
a  
set  
of  
naturals  
of  
the  
form  
{  
1,  
2,  
...  
,  
N  
}  
to  
have  
a  
non-finite  
set  
size.  
The  
naturals  
are  
not  
a  
set  
of  
that  
form  
so  
I  
don't  
see

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you've  
said  
anything  
that  
applies  
to  
the  
set  
of  
all  
naturals.

The  
fact  
is  
true  
for  
all  
 $n$   
in  
 $N$ ,  
that  
the  
set  
up  
to  
and  
including  
any  
one  
of  
them  
is  
finite,  
so  
there  
is  
no  
point  
where  
the  
addition  
of  
any  
elements  
produces  
a  
set  
of  
infinite

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size.

True,  
but  
so  
what?  
The  
naturals  
are  
not  
"produced"  
by  
adding  
elements.

Successor()  
doesn't  
generate  
new  
elements?

No. It doesn't allocate a block  
of memory and then return a  
pointer to  
that memory. All the  
pointers are already defined,  
before you look at  
them.

```
Function Successor(x)
{ Successor(x+1)
}
```

Are you saying this recursive function will  
generate the naturals all  
immediately, or immediately run out of  
memory? I've heard that before. "All the  
elements spring from the axioms all at  
once". That's not very analytical.

Huh? The naturals aren't generated by axioms, they are  
described by the  
axioms. Successor describes a relationship between a given  
natural and  
another.

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What does "not very analytical" mean?

It means its not a mechanical explanation of how the set arises, but a sort of magical \*poof\* it exists. If you want to gauge the size of the set in some infinite case, then you look at how the set grows.

You mean it's axiomatic?

I mean it uses axiomatic method as an excuse not to examine the underlying logic any more than necessary. Axiomatic method has the drawback of giving the illusion that a rule once declared applies in absolutely every circumstance equally. It's very easy to miss hidden assumptions in sets of axioms, and worth examining them in the event of any untoward results.

The identity relation between element count and value characterizes the naturals.

I'm not sure I know what that means. I thought the Peano axioms characterised the naturals? "Identity relation

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between  
element  
count  
and  
value"  
characterises  
sets  
of  
naturals  
of  
the  
form  
{  
1,  
2,  
...,  
N  
}.  
The  
naturals  
are  
not  
such  
a  
set.

Sure  
they  
are.  
You  
can  
start  
anywhere,  
and  
1  
is  
the  
natural  
place  
to  
start,  
especially  
since  
the  
element  
value  
is  
identical  
with  
its

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position  
in  
the  
set.  
It's  
as  
valid  
as  
starting  
at  
0,  
and  
demonstrates  
clearly  
how  
the  
infinitude  
of  
the  
set  
size  
depends  
on  
the  
element  
values.

No,  
the  
naturals  
are  
\*not\*  
a  
set  
of  
the  
form  
{  
1,  
2,  
...,  
N  
}.  
The  
naturals  
do  
\*not\*  
have  
a  
largest

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member.

Every  
element is  
of the form  
"n", and no  
matter  
which of  
those n's  
you look at,  
there is no  
infinite set  
up to that  
point.

Yes, so any set of the form  
{ 1, 2, ..., N } is not the set of  
all  
naturals. I don't disagree.

And every n has such a set associated with  
it.

And the set of naturals is not such a set.

It is the union of all such sets, which are not disjoint, and none of which  
contain an element marking an infinite set.

So? It is still not such a set.

But they are still such elements with such sets of predecessors, none of which  
are infinite, and each a subset of the next.

This in  
sharp  
contrast  
with the real  
interval. No  
matter what  
distinct  
point you

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choose from  
any other  
point, there  
is an  
infinite  
number of  
points  
between  
them.

So the natural numbers don't  
form a continuum, agreed.

Right, they're everywhere sparse, and if they  
contain an infinite number of  
elements, it's because they have infinite  
value range, not because there's any  
point of condensation on the line.

What is the "value range" of the naturals? I can think of only two  
answers : undefined or infinite(UNBOUNDED).

I consider it unboundedly large but finite, because the values in the set of  
differences between naturals from which this range would be chosen can be  
unboundedly large, but are always finite. This is assuming you mean the  
standard naturals.

The finite  
naturals are  
like all  
the points a  
finite  
number of  
points from  
0, compared  
to the  
infinite real  
line  
as the  
interval  
[0,1]. Any  
finite  
number of  
infinitesimal

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points can  
only  
create an  
infinitesimal  
distance. An  
infinite  
number of  
such  
infinitesimals  
produces a  
finite  
distance.  
Likewise,  
an infinite  
number of  
finites  
produces an  
infinite  
distance,  
meaning  
that not all  
points can  
be in finite  
locations.

Seeing as the reals form a  
continuum I'm not sure what  
"a finite  
\*number\* of points from 0  
means". What's the first  
point after 0? If  
it's "x" isn't "x/2" closer to  
0? Shouldn't x/2 have been  
the next  
point, after all? What about  
x/4?

That concept depends on the concept of the  
point as infinitesimal segment on  
the line, as I said. Any infinitesimal  
difference, multiplied by any finite  
number of times, is still infinitesimal.

So what's the first point after 0?

The next nonstandard first level point is Lil'un, or 1/Big'un. In T-riffics,  
it's 0.000...001.

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What about  $0.000\dots 001/2$ ?

That's a subinfinitesimal. In binary T-riffics it would be  $0.000\dots 000:1$  with limit point at  $-\log_2(\text{Big'un})$ . They can be held to exist or not, without breaking Archimedean principle on the finite level. Considering that value adds nothing to standard accuracy. But, subinfinitesimals can be considered. Lil'un<sup>2</sup> is the next nonstandard second level point after 0.

Well,  
then,  
I  
guess  
we  
have  
different  
intuitions.  
Picture  
the  
unit  
interval  
[0,1]  
as  
a  
little  
shrunk-down  
real  
line  
from  
0  
to  
oo.

"to"  
oo?  
The  
real  
line  
doesn't  
have  
endpoints,  
surely?  
But  
[0,1]  
surely  
does.

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<http://mathworld.wolfram.com/AffinelyExtendedReal.html>

<http://mathworld.wolfram.com/ProjectivelyExtendedReal.html>

$-\infty$   
and  
 $\infty$   
aren't  
endpoints  
to  
the  
real  
line  
in  
either  
of  
those.

Affinely  
Extended

–  
"Note  
that  
these  
improper  
elements  
 $\infty$   
and  
 $-\infty$   
are  
not  
real  
numbers,  
and  
that  
this  
system  
of  
extended  
real  
numbers  
is  
not  
a  
field."

How  
can  
something  
which  
isn't

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a  
real  
number  
be  
on  
the  
real  
line?

Uh, well,  
they're  
included,  
aren't they,  
on the  
affinely  
extended  
number  
line?

No. They aren't real  
numbers.

They're included on the line. So, you explain  
to em how something that can;t be  
on the line is there. Oy.

But they aren't included "on the real number line". They  
aren't real  
numbers.

They are included on the Affinely Extende real number line idividually, and  
on  
the Projectively Extended Reals as a mutual point.

No, they aren't. The links you provided don't mention "number lines"  
(actually I can't think of any actual mathematics that talks about  
number lines, it's just a visual aid, right?) but they're very clear  
that are not real numbers. Why then would they be on the \*\*\*real  
number\*\*\* line?

They're not on the standard real line, but they can be added, as you now know.

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Just  
so  
I'm  
clear  
–  
I  
don't  
get  
what  
an  
"infinite  
n"  
is.  
By  
your  
description  
of  
"infinite  
n"  
I  
think  
there  
aren't  
any  
"infinite  
n"  
at  
all.

So  
we're  
back  
to  
finite  
numbers  
of  
points  
in  
the  
line  
segment?  
(sigh)

No,  
we're  
back

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to  
no  
"number"  
of  
points  
in  
the  
line  
segment.  
There's  
not  
finitely  
many  
points.  
So  
I  
say  
in-finitely  
many  
points.  
Infinite  
precisely  
because  
there  
is  
no  
"number"  
of  
points.

No finite  
number.

Why should there be a  
number, at all?

Because there are points, and you can count  
their number. It's just that you'll  
never finish counting, so the number is  
infinite.

Then that's not a "number" in any sense that most people use  
the word.

I guess I'll remind you that "number" has no mathematical  
definition I  
know of.

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It does for me. A number is a symbolic representation of a quantity.

That's a natural language definition of "numeral", not a mathematical definition of "number". If you don't know what a mathematical definition is, we have a problem. How do I determine conclusively if some object is or is not a number?

In your sense of number, or in mine?

At least I'm  
not just in  
hand  
waving  
denial.

Oh, that made me laugh.  
You've been doing nothing  
\*but\* handwaving.

And you've been questioning the very idea  
that a set has a size. What do you  
call that?

Pretty reasonable, without a definition of size? The standard theories have notions of bijectability which match up exactly with the notion of "size" for finite sets and work pretty well for infinite sets. I can agree that any set has a size using the definition of bijectability.

Your definition which is something like "count the elements in the set and the last number you get to is the set size" only gives a size to finite sets. To declare "oh, but you just keep counting until you get to the end of the infinite sequence, and then you've counted to an infinite number, obviously"... that, that is handwaving, Tony.

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If you say so, but some of the simpler ideas involved seem to elude you.

And everyone else on the planet? Weird.

Not to me.

Firstly  
the  
phrase  
"The  
size  
is  
amorphous  
until  
declared  
to  
be  
specific"  
confuses  
me.  
Are  
set  
sizes  
not  
fixed  
in  
your  
theory?  
Does  
the  
set  
of  
naturals  
have  
many  
sizes?

The  
boundary  
between  
finite  
and  
infinite

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cannot  
be  
pinned  
down  
in  
any  
substantial  
sense,  
so  
any  
set  
defined  
using  
finiteness  
as  
any  
criterion  
suffer  
from  
this  
issue.

That  
didn't  
answer  
my  
question.  
Are  
set  
sizes  
fixed  
in  
your  
theory  
or  
not?

Sizes  
of  
infinite  
sets  
are  
comparable  
and  
orderable  
over  
a  
common  
value

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range.  
I  
don't  
know  
what  
you  
mean  
by  
"fixed".  
It  
depends  
on  
the  
value  
range  
you're  
discussing.  
When  
you  
define  
an  
infinite  
set  
with  
a  
recursive  
definition,  
and  
then  
only  
perform  
a  
finite  
number  
of  
iterations,  
it's  
not  
an  
infinite  
set,  
but  
a  
finite  
recursive  
set.  
In  
general,  
one  
will  
compare

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two  
infinite  
sets  
over  
the  
range  
[0, Big'un]  
to  
get  
a  
standard  
measure  
of  
the  
set  
in  
terms  
of  
a  
formula  
on  
infinite  
units.

So  
your  
T-naturals  
\*don't\*  
have  
a  
size?

Yes, over  
the real line  
there are  
Big'un  
T-nats,  
since that is  
the length  
of  
the line in  
unit  
intervals,  
and there's  
one natural  
per unit.  
You  
remember  
the

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identity  
relationship  
between  
element  
count and  
value in the  
naturals?

So BigUn is the maximal  
element of the naturals?  
What about BigUn+1. Is  
BigUn+1 beyond the scope  
of the real line, somehow?

Big'un is the length of the line and the unit  
infinity. On the line, one may  
have as many as  $\text{Big'un}^2$  standard reals.  
Beyond that, one has higher  
dimensional spaces with powers of that  
value. No one said there weren't numbers  
greater than Big'un.

So, again "BigUn" is the number of natural numbers and yet  
you can have  
a natural number greater than "BigUn"?

Big'un is the standard length of the real line, but not the rgeatest number.  
If Big'un is a whole number,  $\text{Big'un}^2$  is too, and bigger than Big'un, being  
the  
number of reals on the line.

What does "the standard length" mean?

The length of the first-order infinite continuum. I guess "standard" was a bad  
choice of word.

What  
is  
"the  
range  
[0, Big'un]"?

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The positive  
real number  
line.

What is the range of  
[0, BigUn+1]?

Big'un+1, greater than the length of the line  
in units.

And what is the length of the line? BigUn! So BigUn is the  
end of the  
line? But you can carry on past it? But I thought it was the  
length of  
the line and the end of the line? Well yes, but you can just  
keep on  
going anyway! So in what sense is it either a length or an  
endpoint?  
Well, don't you see?? It's DECLARED to be so!

Sigh.

It's a UNIT infinity. We count Big'un reals in [0,1). Does that mean 1 doesn't  
exist, or can we have [0,1] with Big'un+1 elements? We can. I never said  
one  
cannot count beyond Big'un. That may seem contradictory to you, but that's  
just  
from years of dealing with aleph\_0. It's not aleph\_0.

Well if BigUn is "the count of unit intervals on the real line" then  
what else is it but the number of counting numbers? You seem to think  
it should be obvious what your BigUn is. It's not. You're the only  
person you've ever explained it to. And I'm not so sure about that.

I told you. It's the unit infinity. It's the number of reals in the unit  
interval and the number of unit intervals on the standard infinite line. It's a  
primitive.

What  
is  
"Big'un"

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for  
that  
matter?  
The  
length  
of  
the  
real  
line?  
What  
does  
"length"  
mean,  
then?  
What  
are  
"infinite  
units"?

Big'un is  
the number  
of reals per  
unit  
interval, and  
the length  
of the real  
line as a  
count of  
unit  
intervals  
and their  
associated  
T-naturals.  
This  
infinite  
number is  
used as a  
unit on the  
infinite  
scale, so  
that we can  
say  
 $\text{Big'un} + \text{Big'un} = 2 * \text{Big'un}$ ,  
just like  
with finite  
units.

What does "length" mean to  
you?

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It means distance between two points,  
measured in some unit intervals end to  
end.

So what point is BigUn at? The end of the line? The end of  
the  
\*unending line\*? The end of the line which you can  
nevertheless carry  
on past just fine?

It's at a standard infinite point which is considered the end of the infinite  
unit interval.

What is "a standard infinite point"?

It's a point on the number line declared to be infinitely many finite units in  
distance from the origin.

1,2,3...Big'un-3,  
Big'un-2,  
Big'un-1,  
Big'un,  
Big'un+1,  
...

What  
on  
Earth  
is  
this?  
"BigUn"  
is  
the  
length  
of  
the  
real  
line,  
but  
you  
can  
have

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"BigUn+1",  
too?  
What  
does  
any  
of  
this  
even  
mean?  
Is  
this  
supposed  
to  
be  
a  
number  
system  
you  
can  
do  
arithmetic  
with?

Yes,  
absolutely,  
and you can  
certainly  
have  
Big'un+1.  
Big'un is  
the size of  
the  
set of  
T-nats, and  
then, say,  
wee throw  
in pi as an  
additional  
member.  
Now we  
have  
Big'un+1  
elements in  
the set.

What are the sizes of the  
following sets?

{0, 1, 2, ...}

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undefined

{0, 1, 2, ... BigUn}

Big'un+1

{0, 1, 2, ... BigUn+1}

Big'un+2

{1, 2, 3, ...}

undefined

{1, 2, 3,... BigUn}

Big'un

{1, 2, 3,... BigUn+1}

Big'un+1

{Pi, 0, 1, 2, ...}

undefined

{Pi, 0, 1, 2, ... BigUn}

Big'un+2

{Pi, 0, 1, 2, ... BigUn+1}

Big'un+3

{Pi, 1, 2, ...}

undefined

{Pi, 1, 2, ... BigUn}

Big'un+1

{Pi, 1, 2, ... BigUn+1}

Big'un+2

So the Tnaturals have a different size depending on whether you start from 1 or 0? So you can't define any kind of size for sets without a

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maximal element? So the largest natural BigUn nevertheless has a natural larger than it? How quaint.

Show me where I said Big'un is the largest counting number. It's the number of unit intervals on the line, but you can count more points than the whole numbers. It's a unit infinity.

You can count more than the counting numbers? With what?

More counting numbers. Do you really expect to find an end to infinite sets, or does infinity have multiple levels? The concept of counting beyond the finites is not unreasonable.

You honestly think there are  
"more" element in the  
sequence  
pi, 0, 1, 2, ....  
Than  
0, 1, 2, 3, ....

Without a maximal element there is no value range over which to compare the sets, but given the same value range, yes, the first has one more element.

These are sequences not sets. What does "value range" mean here?

it means that the set contains one more element between any two given values than the other, as long as the value range includes pi.

But these are sequences not sets. What does "value range" mean here? And why "between two given values"? I don't care about sequences that end, here. I'm interested in the ones that go on forever.

I know what you're interested in. Proving that what you already know is better than anything anyone can invent. That's nice. I'm trying to prove we can do

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better than what we have. See below.

What about

2, 4, 6, 8, 10, 12, ...

1, 2, 3, 4, 5, 6, ...

Over the same value range, the first has half as many elements.

What about over the same range of the sequence? First 10 terms? 10 terms each. First 100 terms? 100 terms each. First n terms? n terms each. First n+1 terms? n+1 terms each! First BigUn terms? BigUn terms each! First BigUn+1 terms? BigUn+1 terms each!

Right, that's the standard treatment. You look only at corresponding elements without regard to how quickly each progresses along the line. What I am suggesting is that if you look at the number of elements within a given value range, that gives a measure of the density of the set on the line. Unless one of the sets has greater range than the other, the set with the greater density should be considered the larger set.

What line? These are sequences, not sets. I'm not asking about density. I don't care about density. If I wanted to determine the density within a value range I could do so. Density doesn't mean anything without a "value range", so it seems completely fruitless to demand that sequences without an end or sets without a value range should be compared in terms of things they are not.

Yes, Tony, IF these sequences ended at some certain value then one of them would be longer than the other. But they don't. They go on \*forever\*. Can you really not get your head around that? That sometimes we want to talk about things that \*don't\* have a value range?

$$\text{range} * \text{density} = \text{size}$$

$$\text{units} * (\text{elements} / \text{unit}) = \text{elements}$$

Wrap your head around it, Mike.

## Re: Calculus XOR Probability

"In the infinite case" the "inductively proven equality" between "the number of terms" of sequence A and sequence B still holds!

Yes, in the infinite case of the set index, but not when set density is taken into account.

Who mentioned density? Not me. I asked about set sizes. What does density have to do with set sizes? Isn't set size the number of elements?

Yes, the size is the number of elements, and the average density of a set of reals on the real line, in elements per unit, multiplied by the value range of the set, in units, gives that size, in elements. When one set is denser than another in every unit interval over the line, that is, has more elements in every chosen interval, then isn't it obviously larger? Seems so to me. :)

Not at all. It leads to very intuitive and consistent conclusions. It all depends on what you consider "size". Cardinality doesn't fit the bill when it comes to infinite sets.

The idea that the sequences  
0, 1, 2, 3, ...  
1, 2, 3, 4, ...  
2, 3, 6, 8, ...

have different "lengths" is

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neither intuitive not  
consistent.

Even though the first is a proper superset of  
the others? Hmmm....

These are sequences not sets, Tony! 0, 1, 2, 3 is not a  
supersequence(?)  
of 2,4,6,8.

Look tony, can't you see? The inductively proven equality  
between terms  
of sequences holds even in the infinite case. You keep on  
counting and  
never run out of numbers for any of the sequences, even  
when you reach  
the  $\text{BigUn}^{\text{th}}$  term. Then the first sequence is at  $\text{BigUn}-1$  the  
second is  
at  $\text{BigUn}$  and the third has gotten to  $2*\text{BigUn}$ . And you can  
keep on  
going!

Yes, \*I understand the standard half-baked analysis.

Hahahahaha. And your analysis is super rigorous? Not contradictory at  
all? Doesn't rely on "magically declaring things to exist"? Hahaha. At  
least standard theories explain all of their terms without having to  
resort to natural language handwaving whenever probed too deeply.

Perhaps it's been so long etched in stone that it's fully understood, and it's  
time to move on to something more fruitful. So, while things develop, they're  
not yet etched in stone. Sue me. :)

—  
Smiles,

Tony

mike.

Re: Calculus XOR Probability

—  
Smiles,

Tony

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