Re: How to prove that a random sort algorithm converges?

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* *Date*: Thu, 18 May 2006 12:01:35 +0000 (UTC)

On Wed, 17 May 2006 21:39:32 −0400, Tim Peters wrote:

> [James]
>
> > Thanks to all who contributed in one way or another.
> > Although I will need sometime to digest your suggestions, there are a few things I learn at the moment:
> >
> > (1) There are more than one way to prove the algorithm.
>
> That's true. The way I suggested is the easiest of all those I saw, although it also gives the least information.
>
> > (2) It appears that saying "the algorithm converges is not precise enough". We need to define what is the convergence criterion.
>
> Well, "converges" has no conventional meaning here at all, so you have to define what you mean. Convergence usually applies to iterative numerical algorithms, such as talking about how quickly a root−finding algorithm can be expected to converge to a root to within a given tolerance.

What's wrong with pointwise convergence? It looks like a perfectly standard example of convergence to me.

In fact, since there can be only a finite number of transpositions, it is a certainty that the state must converge. However, we need to allow for the possibility of convergence to something other than the sorted state, since a needed comparison might never be made. The probability of this happening is 0, but it is nonetheless possible. The distinction between "convergence" and "convergence to the sorted state" is one that I overlooked previously.

> It appears that what you want to know is whether your algorithm _terminates_, but there was no termination criterion given. People are reasonably guessing that you intend for your algorithm to terminate when the list reaches a wholly sorted state.

It's not necessary to speak of "termination". All we need is for the state to become eventually constant, which it certainly does. A constant sequence of states converges pointwise.

> > (3) It also appears that saying "the algorithm converges with probability 1" is better than saying "the algorithm will converge".

> Not really, unless _you_ define "convergence" in terms of probability.

Note the difference betweeen "convergence" and "convergence to the sorted state." The former is a mathematical certainty, while the latter merely occurs with probability 1.

Consider the sequence "1 3 2 4". In order for the sequence to become sorted, it is necessary that the middle pair of numbers be selected for comparison at some point. We cannot prove that this comparison will ever be selected, but we can show that it will eventually be selected with probability 1.

However, whether the central pair is ever selected or not, we can say with certainty that the sequence converges. It may converge to "1 3 2 4" (which happens with probability 0), or it may converge to "1 2 3 4" (which happens with probability 1), but it most definitely converges to *something*.

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Dave Seaman
U.S. Court of Appeals to review three issues
concerning case of Mumia Abu−Jamal.
<http://www.mumia2000.org/>
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