Re: Calculus XOR Probability

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- *From*: cbrown@xxxxxxxxxxxxxxxxx
- *Date*: 20 May 2006 00:17:22 −0700

Tony Orlow wrote:

> Matt Gutting said:
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> > Tony Orlow wrote:
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> > > Virgil said:
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> > > > In article <MPG.1ed290581a4f392198acd4@xxxxxxxxxxxxxxxxxxxxxxxxx>, Tony Orlow <aeo6@xxxxxxxxxxx> wrote:
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> > > > > cbrown@xxxxxxxxxxxxxxxxx said:
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> > > > > > Tony Orlow wrote:
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> > > > > > > For the last time, no. If the limit of the staircase is anything different from the diagonal, which it is,

then
there
is
no
contradiction.

There is no mathematically valid model in which the limit of the sequence of staircase functions is anything but the diagonal function.

If TO wished to claim otherwise, then he must create and present to us the entire system in which he claims his allegations hold, as they do not hold in any current system.

Okay. Here goes.

Rather than a set of points, let us define both the staircase and the diagonal as sequences of segments ....

In other words, rather than saying "there is no natural number x such that 2*x = 3", let us say "there is no rational number x such that 2*x = 3".

Does it surprise you that these two statements provide different answers?

... defined as a pair of reals which represent the x and y coordinate differences between subsequent points. Let us compare the two thus in a segment−wise manner, maintaining the same number of segments in each, and see if the segments which describe the staircase approach those that describe the diagonal. Where n=1, we have two segments to the staircase, {0,1} and {1,0}, for a total change of {1,1}. Dividing the diagonal into two segments we have {1/2,1/2} and {1/2,1/2}, also for a total change of {1,1}. Now, as n increases we have {1,1}=sum(x=1−>n: {1/n,0}+{0,1/n}) for the diagonal, and sum {1,1}=(x=1−>n: {1/2n,1/2n}+{1/2n,1/2n}) for the diagonal. While the locations of the points in each segment become arbitrarily close, the

> vectors defining
> the segments of which the lines are made never become close, but are always at
> a 45 degree angle to their corresponding segments in the other line.
>
> When you look at the distance traveled, you sum all the x components of the
> vectors in each line and sum all the y components, and you get {1,1} in both
> cases, and the distance is sqrt(2).
>
> When you look at the lengths of each, you sum the length of each vector in the
> line. For the staircase we have sum(x=1−>n: 1/n+1/n)=2. For the diagonal we
> have sum(x=1−>n: 1/sqrt(2)+1/sqrt(2))=sqrt(2). Because of the difference in
> vector direction, even at the infinitesimal scale, the staircase is longer than
> the diagonal.
>
> Is that an "entire" enough "system" for you? :D

No, because all that I see you have done is to note (using unnecessarily obfuscatory language) that the limit of the length of the staircases is 2; which no one is disagreeing with; and that the length of the diagonal is sqrt(2), which also no one is disagreeing with.

What is missing is a statement of /exactly what you mean/ by "the length of (the limit of the staircases) is {whatever you propose}".

In order for me to understand your answer, you must first state /exactly what you mean/ by "the limit of the staircases"; which you have not done in the above paragraphs. Is "the limit of the staircases" a function? Is it a real number? A set of line segments? A set of pairs of pairs in R^2 x R^2? A white elephant?

The closest you get is this cryptic comment: "Because of the difference in vector direction, even at the infinitesimal scale, the staircase is longer than the diagonal." But this doesn't tell me what "the limit of the staircases" is; it simply mentions several (undefined) properties you propose it to have.

For example, presumably there is some point p = (a,b) in R^2 that is in the limit of the staircases. Does that point satisfy b = 1 − a, or does it not?

Given that point p, what is the "vector direction, at the infinitesimal scale" associated with it? Can we deduce it from the values of a and b? For example, how do I determine the "vector direction, at the infinitesimal scale" at the point (1/2,1/2) (which I presume is in the "limit of the staircases")?

Given two points p and q in R^2 which are in the limit, how do I determine whether p and q have the same or different "vector directions, at the infinitesimal scale"?

Once you have addressed these questions, we can suppose that your definition of "the limit of the staircases" is a mathematical object called "L". /Then/ I can evaluate a statement you might make of the form "the length of L is {whatever you propose}".

Until then, you haven't defined what you mean by "the length of (the limit of the staircases)"; all you have defined is "the limit of (the length of the staircases)"; and at least in its result, we are all in agreement: the limit of the length of the staircases is 2, and the length of the diagonal is sqrt(2).

The remainder of your "definition" leaves me as desirous of a definition as before: you have simply introduced new, undefined terms to define a previously undefined term. This renders your definition no more meaningful than it was before, mathematically speaking.

<snip>

> > First of all, it's not that "the points become the same set in the limit". It's that the limits of the two sets of points are identical (the same set of points). Nothing "becomes anything in the limit".

> Your objection is semantic? Take it to alt.picky.english.

No, his objection is that you imply that for each point in the limit, there is some /unique/ continuous /curve/ of points which can be identified as "becoming the point in the limit".

That is not a feature of the definition of "limit" I gave (nor is it, in general, a feature of various other definitionsof "limit" I have seen).

It does not require a /unique/ sequence to be identified with each point in the limit; simply that /at least one/ such sequence exists for a point to be considered a limit point of the sequence of sets of pairs in R^2.

Nor does it generate a continuous /curve/ of points which is associated with a particular point in the limit; it provides a discrete /sequence/ of points which converges to a point in the limit.

> > Second, the statement Virgil is making is not a leap; it's a consequence of the definition of limit. Unless you have a different definition.

> I just offered one that explains the discrepancy. Did you read any of it? Is this supposed to explain why my limit definition "doesn't make sense", as you claimed in your next post to have shown? Nice hand waving.

Aside from the fact that you have not even provided a definition of what kind of mathematical object "the limit of the staircases" is (a set? a real number? an equation?), I don't see how your discussion of limit above applies to anything that is not a collection of segments; which is to say your definition is (at best) simply providing an example, not providing a proper definition.

For example, let C_n = {(a,b): b = sin(n*a)/n^2}. According to my definition of limit, lim n−>oo {C_n} = {(a,b): b = 0}, as you should be able to see for yourself by applying my definition.

Could you walk us through how your definition of limit applies to this sequence? What do you claim you mean by "lim n−>oo {C_n}" in this case? Is it a set? Is it a function? Is it a real number?

Cheers − Chas

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