Re: Calculus XOR Probability

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- *From*: Tony Orlow <aeo6@xxxxxxxxxxx>
- *Date*: Tue, 23 May 2006 10:20:42 −0400

Matt Gutting said:

> Tony Orlow wrote:
>
>> cbrown@xxxxxxxxxxxxxxxxx said:
>>
>>> What is missing is a statement of /exactly what you mean/ by
>>> "the
>>> length of (the limit of the staircases) is {whatever you
>>> propose}".
>>
>> The limit of the staircases is the series Sum(n−>oo: {1/n,0},{0,1/n}). That's n repetitions of a step with length 2/n, for a total length of 2.
>
> You're assuming again that you can interchange the sum and the limit process. The length of the limit of staircases need not equal the limit of the length of staircases with the standard definition.
>
> Do you mean to say that the limit of the staircases is a series? That's how your sentence is phrased, but it doesn't seem to make sense. You're apparently making a sequence of geometric figures (staircases), then stating that the limit is an infinite series, presumably evaluated in the same way that infinite series typically are − although you need to be clearer about the meaning of "n−>oo (1/n,0),(0,1/n)". How does the limit of a sequence of geometric figures get to be a sequence of real numbers? Or is that what you meant?

I guess that wasn't exactly clear. I wasn't sure what notation I should use. When I say "Sum(n−>oo: {1/n,0},{0,1/n})", I should have said "lim n−>oo sum(x= 1−>n: length({0,1/n})+length({1/n,0})". I mean that each stair consists of the two pairs, one denoting the tread and the other the riser, the curvilinear length being the sum of their lengths, which is 2/n. So, the overall length becomes lim n−>oo sum(x=1−>n: 2/n)=2.

For each of these pairs of the form {1/n,0} and {0,1/n} are corresponding segments in the diagonal of form {1/(2n), 1/(2n)}, each with a length of sqrt(2)/(2n), for a total length of sqrt(2)/n, compared to 2/n for each stair. The diagonal is lim n−>oo sum(x=1−>n: sqrt(2)/n)=sqrt(2).

> > > In order for me to understand your answer, you must first
> > > state
> > > /exactly what you mean/ by "the limit of the staircases";
> > > which you
> > > have not done in the above paragraphs. Is "the limit of the
> > > staircases"
> > > a function? Is it a real number? A set of line segments? A set
> > > of pairs
> > > of pairs in R^2 x R^2? A white elephant?

> > I stated already it's a sequence of line segments. See above, "defined as a pair of reals which represent the x and y coordinate differences between subsequent points". Each of those pairs represents a line segment.

> So, each staircase is a sequence of line segments. How do you decide that the limit is also a sequence of line segments?

Because that's the way the line is defined. So, you think your line is a set of points. How do you know the limit is also a set of points? What is the point of this question? What do you THINK the sequence of segments becomes? Ask Newton.

> > > The closest you get is this cryptic comment: "Because of the
> > > difference
> > > in vector direction, even at the infinitesimal scale, the
> > > staircase is
> > > longer than the diagonal." But this doesn't tell me what "the
> > > limit of
> > > the staircases" is; it simply mentions several (undefined)
> > > properties
> > > you propose it to have.

It's a staircase with oo stairs, each 2/oo long, given riser and tread. What is your question?

My question is, since you haven't actually defined oo, how can you tell whether oo or 2/oo exist?

Because that's the LIMIT. You want to take the limit as n−>oo? Well, oo has to exist, doesn't it? You have a "taxicab" distance of 2? It doesn't matter WHAT rectilinear approaching path you take, it'll always be 2. So, if you think the limit of the staircase DOESN'T have a length of 2, it's not a taxicab distance, and the object is no longer a staircase. If it's still a staircase, with an infinite number of infinitesimal stairs, the length IS 2, because that's the nature of the staircase. In any case, you're talking about the limit as n−>oo, so what makes YOU think oo exists?

Of course, you asked a different question from last time, so I am not sure you know WHAT you're asking. The limit of the staircase is a staircase in the limit. The difference between the diagonal and the staircase cannot be distinguished by location alone. By defining the curve as a sequence of segments, rather than a set of locations, the difference is quite detectable, because the segment definition preserves the notion of direction IN THE LIMIT.

For example, presumably there is some point p = (a,b) in R^2
that is in
the limit of the staircases. Does that point satisfy b = 1 − a, or
does
it not?

The tread of one step meets the riser of the next at a point on the diagonal. Where the riser meets its tread, that corner is NOT on the diagonal, even if it may be only an infinitesimal difference away, and consider coincident with
the
line according to stard finitist limits.

Given that point p, what is the "vector direction, at the
infinitesimal
scale" associated with it? Can we deduce it from the values
of a and b?
For example, how do I determine the "vector direction, at the
infinitesimal scale" at the point (1/2,1/2) (which I presume is
in the

"limit of the staircases")?

The point (1/2,1/2) is in every staircase for n>1, for sure. The direction of the tread before it is horizontal, and the diretcion of the riser after that point is vertical. Remember, directions are not defined for points, but for segments. That point has not direction of its own, hence the need to look at the limit, not of the points, but of the segments.

How do you know that the limit of the segments exists, and that it is a segment?

Because that's the way it's defined, whether as a starting point and a vector, or two endpoints. When the points or offsets are infinitesimal, the locations may be indistinguishable, but the direction is not.

Given two points p and q in R^2 which are in the limit, how do I
determine whether p and q have the same or different "vector directions, at the infinitesimal scale"?

Points do not have directions, ultimately. The segment {1/2,0} is horizontal, and {0,1/2} is vertical.

Okay, so how about the infinitesimal scale?

{0,1/n} is still vertical, and {1/n,0} horizontal, even if n is infinite. Those 0's are absolute 0's. There is no horizontal change in any riser, or vertical change in any tread. the 1/n's have a limit of 0 as n−>oo, but what that essentially means is that, for any given actual infinite n, 1/n is infinitesimal, and larger than absolute 0. Direction is maintained.

Once you have addressed these questions, we can suppose that your
definition of "the limit of the staircases" is a mathematical object

called "L". /Then/ I can evaluate a statement you might make
of the
form "the length of L is {whatever you propose}".

Are you sure you won't ask the alreayd answered questions, again?

I still have questions about your answers to the questions.

Just as long as they're not the same questions that I already answered, or
we're just going around in circles, which I suppose serves some purpose anyway,
but seems rather like a waste. Anyway, carry on....

Until then, you haven't defined what you mean by "the length
of (the
limit of the staircases)"; all you have defined is "the limit of
(the
length of the staircases)"; and at least in its result, we are all
in
agreement: the limit of the length of the staircases is 2, and
the
length of the diagonal is sqrt(2).

But you disagree that the limit of the staircases is anything other than the
diagonal, whereas I have demonstrated a form of limit which shows clearly
that
there's a difference, and which accounts precisely for the error.

I don't see a clear definition of limit. Can you fill in the blanks here:

DEFINITION: The limit of a ____(insert name of mathematical object)
is a ___ (insert name of a mathematical object) satisfying the following
criteria: _______.

The limit of a curve is curve satisfying the following criteria:

A curve is defined as a series of pairs {x,y}, the first denoting the x and y
offset of the first point from the origin in R^2, and each subsequent pair
being the offset of the next point from the last.

The offsets are defined with a formulaic relation to the number n of points defined, such that knowing n and the relation, one can specify each offset which defines the curve.

The limit as n−>oo is defined to be the infinite sequence of xy offset pairs which are each the limit of the xy pairs as defined by the relation for any n.

I think this last part is missing a little something, but you'll probably point that out.

> Both blanks have to be filled with terms which either are agreed upon generally, or are defined in turn according to the template provided.
>
> Once you can fill in those blanks, then we have something we can talk about. Until then, your definition is not sufficiently well−formed to be able to discuss anything related to it.

Your serve.

−−
Smiles,

Tony
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