Re: Calculus XOR Probability

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- *From*: Tony Orlow <aeo6@xxxxxxxxxxx>
- *Date*: Wed, 24 May 2006 10:38:38 −0400

Matt Gutting said:

> Tony Orlow wrote:
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> > > > Matt Gutting said:
> > > >
> > > > > *My* question is, since you
> > > > > haven't actually defined oo,
> > > > > how can you tell
> > > > > whether oo or 2/oo exist?
> > > >
> > > > Because that's the LIMIT. You want to take
> > > > the limit as n−>oo?
> > >
> > > Yes, or writing it out without shorthand, I want to take the
> > > limit as
> > > n increases without bound.
> > >
> > > > Well, oo has to
> > > > exist, doesn't it?
> > >
> > > Not necessarily.
> >
> > Oh. Then the symbol doesn't necessarily mean anything. Can you take a limit
> > as
> > n approaches soemthing that doesn't exist?
>
> No, and I'm not. It's not true that "n approaches infinity"; n increases without
> bound. And one can certainly take a limit as n increases without bound.

Right, but you can't say what the curve IS in the limit without considering
having REACHED the limit.

> > > You have a "taxicab" distance of 2? It
> > > doesn't matter WHAT
> > > rectilinear approaching path you take, it'll
> > > always be 2. So, if you think the
> > > limit of the staircase DOESN'T have a
> > > length of 2, it's not a taxicab distance,
> > > and the object is no longer a staircase.
> >
> > That's exactly it. There's no requirement that the limit object
> > be the same sort
> > of thing as the members of the sequence.
>
> You don't require it, perhaps, but then again, if you think they are the same,
> then what happened to your arclength measure?

> > > If it's still a staircase, with an
> > > infinite number of infinitesimal stairs, the
> > > length IS 2, because that's the
> > > nature of the staircase. In any case, you're
> > > talking about the limit as n−>oo,
> > > so what makes YOU think oo exists?
> >
> > I'm not talking about the limit as n approaches anything, as
> > you seem to imply
> > from the way you write "the limit as n−>oo". I'm talking
> > about the limit as n
> > increases without bound. I don't believe oo exists as a
> > number.
>
> Then you have no business talking about the identity between the staircase
> "in
> the limit" and the diagonal. If oo doesn't exist, then they never are the same,
> and the whole discussion goes out the window.

See my reply above.

The staircase approaches the diagonal, location−wise, but in the end, it's not the same object, as I've demonstrated.

> > > > Of course, you asked a different question from last time, so I am not sure you know WHAT you're asking. The limit of the staircase is a staircase in the limit.
> > >
> > > Can you prove that assertion?
> >
> > I have demonstrated a concept of limit that shows it. while the treads and risers become infinitesimal, their direction never changes, and never approaches the direction of the diagonal.
>
> You haven't defined "infinitesimal" to anyone's satisfaction, certainly not to mine.

Any finite divided by any infinite yields an infinitesimal. If you divide the unit interval by n, as n−>oo, the subintervals have lengths approaching 0. For any specific, non−absolute infinity, you have an infinitesimal value. As the number increases without bound, the subintervals shrink with a bound of 0, but never ultimately reach that bound, as n never ultimately reaches absolute oo. But, perhaps you just aren't partial to infinitesimals.

> > > > The difference between the diagonal and the staircase cannot be distinguished by location alone. By defining the curve as a sequence of segments, rather than a set of locations, the difference is quite detectable, because the segment definition preserves the notion of direction IN THE LIMIT.
> >
> > See?
>
> No; you're assuming that the limit is some construct involving infinitesimals.

I am assuming that when n is a specific infinity, 1/n is a specific infinitesimal, and that's part of the problem we're discussing.

For example, presumably there is some point p = (a,b) in R^2 that is in the limit of the staircases. Does that point satisfy b = 1 − a, or does it not?

The tread of one step meets the riser of the next at a point on the diagonal. Where the riser meets its tread, that corner is NOT on the diagonal, even if it may be only an infinitesimal difference away, and consider coincident with the line according to stard finitist limits.

Given that point p, what is the "vector direction, at the infinitesimal scale" associated with it? Can we deduce it from the values of a and b? For example, how do I determine the "vector direction, at the infinitesimal scale" at the point (1/2,1/2) (which I presume is in the "limit of the staircases")?

The point (1/2,1/2) is in every staircase for n>1, for sure. The direction of the tread before it is horizontal, and the diretcion of the riser after that point is vertical. Remember, directions are not defined for points, but for segments. That point has not direction of

its own,
hence the
need to look
at
the limit,
not of the
points, but
of the
segments.

How do you know that the
limit of the segments exists,
and that it is a
segment?

Because that's the way it's defined, whether
as a starting point and a vector,
or two endpoints. When the points or offsets
are infinitesimal, the locations
may be indistinguishable, but the direction is
not.

If the locations are truly indistinguishable, then the endpoints
are identical,
and the result is a point, not a segment.

Incorrect. Even an infinitesimal is larger than absolute 0, so n/n is not 0/0,
but 1, and n/0 is still infinite, even when n is infinitesimal. So, the risers
are still vertical, not diagonal. Infinitesimal differences are not equalities.

If an infinitesimal is larger than 0, one can distinguish between one endpoint
and the other one an infinitesimal distance away, no?

Yes, on the infinitesimal scale, not on the finite scale, as in standard
mathematics.

Given
two
points
p
and
q
in

R^2
which
are
in
the
limit,
how
do
I
determine
whether
p
and
q
have
the
same
or
different
"vector
directions,
at
the
infinitesimal
scale"?

Points do
not have
directions,
ultimately.
The
segment
{1/2,0} is
horizontal,
and {0,1/2}
is vertical.

Okay, so how about the
infinitesimal scale?

{0,1/n} is still vertical, and {1/n,0}
horizontal, even if n is infinite. Those
0's are absolute 0's. There is no horizontal
change in any riser, or vertical
change in any tread. the 1/n's have a limit of
0 as n−>oo, but what that
essentially means is that, for any given
actual infinite n, 1/n is
infinitesimal, and larger than absolute 0.
Direction is maintained.

Once you have addressed these questions, we can suppose that your definition of "the limit of the staircases" is a mathematical object called "L". /Then/ I can evaluate a statement you might make of the form "the length of L is {whatever you propose}".

Are you sure you won't ask the alreayd answered questions, again?

I still have questions about
your answers to the
questions.

Just as long as they're not the same questions that I already answered, or
we're just going around in circles, which I suppose serves some purpose anyway,
but seems rather like a waste. Anyway, carry on....

Until then, you haven't defined what you mean by "the length of (the limit of the staircases)"; all you have defined is "the limit of (the length of the staircases)"; and at least in its result, we are all in agreement: the limit of the length of the staircases is 2, and the length of the diagonal is sqrt(2).

But you disagree that the limit of the staircases is anything other than the diagonal, whereas I have demonstrated a form of limit which shows clearly that there's a difference, and which accounts precisely for the error.

I don't see a clear definition of limit. Can you fill in the blanks here:

> DEFINITION: The limit of a ____(insert name of mathematical object) is a ___ (insert name of a mathematical object) satisfying the following criteria: ______.

The limit of a curve is curve satisfying the following criteria:

A curve is defined as a series of pairs {x,y}, the first denoting the x and y offset of the first point from the origin in R^2, and each subsequent pair being the offset of the next point from the last.

The offsets are defined with a formulaic relation to the number n of points defined, such that knowing n and the relation, one can specify each offset which defines the curve.

The limit as n−>oo is defined to be the infinite sequence of xy offset pairs which are each the limit of the xy pairs as defined by the relation for any n.

I think this last part is missing a little something, but you'll probably point that out.

> Both blanks have to be filled with terms which either are agreed upon generally, or are defined in turn according to the template provided.
>
> Once you can fill in those blanks, then we have something we can talk about. Until then, your definition is not sufficiently well−formed to be able to discuss anything related to it.

Your serve.

By your definition of "curve", the set of points {(0,0),(1,1)}
is a curve.
Is that intentional?

By my definition, the segment between them is a curve. When defining a curve
simply as a sequence of points, a two point sequence is allowed, as two points
determine a line. For a continuous curve, one can apply Archimedean principle
to the sequence, but sparse curves are not unheard of. Look at a graph of the stock market. It's a sparse curve, down to the minute, or hour, not the moment.

Generally, my understanding of curves is that they're defined almost everywhere. I'll check up on this.

That's the common notion of a curve, but a general definition may be adopted that distinguishes between continuous and discrete curves. What exactly do you call a series of points separated by finite space? It can be considered a discrete curve, and Archimedean principle may be applied to get the common notion of curves as continuous.

It looks as if by your sentence about "formulaic relation" you mean something
like "the offsets (x_n,y_n) are determined by a function whose domain includes
n, and which doesn't change for any n". Is that a correct interpretation?
Does the function need to be specifiable by a formula, or can it be a list
of input−output values? If it can't, why not?

If you want to find the limit of the curve as n−>oo, then you need to be able to specify the segments using a formula of some sort, because you can't list all of an infinite set of pairs. It must be parameterized with n, in order to find a limit as n−>oo. Do you see any other way?

When you say, "the offsets (x_n,y_n) are determined by a function whose domain

>> includes n, and which doesn't change for any n", that seems a little off. Let's say A n e N A m e N m<=n −> E {x_mn, y_mn} such that x_mn=f_x(m,n) and y_mn=f_y
>> (m,n). In other words, where the curve includes n points there are n segments (including initial offset form the origin) each defined as offsets {x_mn, y_mn}
>> which are calculated using f_x and f_y based on the position in the sequence, m, and the length of the sequence, n. Does that clear things up a little?

> Okay, that's better. I think.

Whew! ;)

>>> By "the limit as n−>oo", I assume that you mean "the limit *of a sequence
>>> of curves C_n* as n−>oo". I'm trying to figure out what you mean by the
>>> sentence though. The n *could* refer to an indexed curve in the sequence
>>> of curves, or to an indexed point on a specified curve. Or, I suppose,
>>> it's possible it might refer to something else. Clarification?

>> My pleasure. The variable n here denotes the number of segments in the curve.
>> Each of those segments has a position in the sequence, m, from 1 through n (including the initial offet). Each n, or number of segments, denotes a different curve, and as n−>oo and the number of segments increases without bound, we have the "curve in the limit". What groups all these curves together
>> as one family is the pair of formulas that give the offsets in each segment of the curve, f_x and f_y. Did that help clarify things? :)

> To an extent. I was indexing curves by their position in the sequence, which was coincidentally the number of segments; so that agrees with what you're saying. But I'm still unclear about what you mean by "we have the 'curve in the limit'." Is the "curve in the limit" one of the curves in your sequence of curves?

Sure, if we consider n from 1 through Big'un, we can sonsider the value at

Big'un to be the value in the limit.

> If so, at what position can it be found?

Big'un.

> Are there curves beyond it in the sequence? What do they look like?

Well, where the staircase becomes infinitesimally close to the diagonal, there is still distance between them at points. One CAN continue to apply the formula regarding the staircase to greater values than Big'un, in which case one is going into the subinfinitesimal range of differences. While this process can continue, getting closer than even a first order infinitesimal distance, the directions continue to be different between the two at every point in the curve.

> Alternatively,
> is it produced from the curves of the sequence by some process or mechanism?
> If so, how is it produced?

When the curve is defined by the pairs denoting the x and y offsets produced by each segment of the curve, using a formula for the offsets based on the number of segments and the position of each segment, then you can take that formula to the limit as the number of segments approaches oo. If the limits of two curves defined this way are equal, then they are the same, and will have the same measure.

> Matt

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Smiles,

Tony
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