Re: naive question from a non−mathematician

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- *From*: David C. Ullrich <ullrich@xxxxxxxxxxxxxxxx>
- *Date*: Sun, 28 May 2006 13:29:08 −0500

On Sun, 28 May 2006 19:21:22 +0200, Denis Feldmann
<denis.feldmann.asupprimer@xxxxxxxxxxxxxxxx> wrote:

> David C. Ullrich a écrit :
>
>> On 27 May 2006 20:42:18 −0700, "Gene Ward Smith"
>> <genewardsmith@xxxxxxxxx> wrote:
>>
>>> Stephen Montgomery−Smith wrote:
>>>
>>>> Like I said in a different post, it depends
>>>> upon your framework. Most
>>>> modern mathematicians define everything in
>>>> terms of sets and set theory.
>>>
>>> Most modern mathematicians define things up to
>>> isomorphism.
>>>
>>>> Thus the natural numbers are, more or less,
>>>> defined as the set of
>>>> finite ordinals, the integers are pairs of
>>>> natural numbers quotiented
>>>> out by the equivalence relation (a,b)~(c,d) iff
>>>> a+d=b+c, the rationals
>>>> are pairs of integers (the second being
>>>> non−zero) quotiented out by
>>>> another approprate equivalence relation, the
>>>> reals are constructed from
>>>> the rationals usually either by Dedekind
>>>> sections, or by some quotient
>>>> of the cauchy sequences, and the complex
>>>> numbers are pairs of real numbers.
>>>
>>> If you like, you can do things this way. Nobody forces you
>>> to, and you

> > could do it other ways.

> Surely there are many ways of setting up all these things. But:

> > You could, for instance, define the reals
> > axiomatically,

> For example the reals are a complete ordered field. This raises the question of how we know that there exists a complete ordered field; if there is no such thing then the theory is a little boring.

> > and recover the rationals and integers from that.

> > > There are just so many ways of defining
> > > these objects in set theory, and
> > > so to say R is a subset of C just doesn't cut it
> > > if you are going to be
> > > nit−picking.

> > Let's say I define the complex numbers C as an algebraically
> > closed
> > field of characteristic zero and cardinality the continuum
> > with a
> > distinguished automorphism conj(z) of degree two.

> Same question here.

Moreover, its not a univoque definition : take a non−standard extension of C with same cardinal (for instance, the quotient of C^N by the classical* relation induced by a (non−trivial) ultrafilter on N)

* This is (z_k)~(z'k) <=> {k/ z_k=z'k} \in U , U being the ultrafilter.

> > Now I define the real
> > numbers R as the subextension fixed by this automorphism,
> > which I now
> > dub "complex conjugation".

> > > > This is R* (the same non−standard extension of R)
>
> > > > Now I define an archimedian absolute value
> > > > by |z| = sqrt(z conj(z)).
>
> > > Why archimedian????
>
> > > > Now C and R are topological fields under the
> > > > topology defined by this, and conj is continuous. Now I
> > > > define Q as the
> > > > intersection of all subfields of C (or R.)
>
> > > There is a much simpler way : Q is the field generated by 1.
>
> > > > Z is the ring of integers of
> > > > Q. I've defined things so that, by definition, Q is a subfield of
> > > > C. Of
> > > > course doing it your way I have a Q in C which is uniquely
> > > > isomorphic
> > > > to the original Q which was constructed, and so forth blah
> > > > blah.
>
> > Serious question: If you start with a structure satisfying your
> > definition of C and then do all this, does it follow that the
> > R you get is complete (in the sense that every nonempty set
> > bounded above has a least upper bound?) I bet it doesn't.
>
> Sure : it is not even archimedian :−)

That's what I thought.

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David C. Ullrich
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