Re: naive question from a non−mathematician

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* *From*: "Gene Ward Smith" <genewardsmith@xxxxxxxxx>
* *Date*: 27 May 2006 20:42:18 −0700

Stephen Montgomery−Smith wrote:

> Like I said in a different post, it depends upon your framework. Most modern mathematicians define everything in terms of sets and set theory.

Most modern mathematicians define things up to isomorphism.

> Thus the natural numbers are, more or less, defined as the set of finite ordinals, the integers are pairs of natural numbers quotiented out by the equivalence relation (a,b)~(c,d) iff a+d=b+c, the rationals are pairs of integers (the second being non−zero) quotiented out by another approprate equivalence relation, the reals are constructed from the rationals usually either by Dedekind sections, or by some quotient of the cauchy sequences, and the complex numbers are pairs of real numbers.

If you like, you can do things this way. Nobody forces you to, and you could do it other ways. You could, for instance, define the reals axiomatically, and recover the rationals and integers from that.

> There are just so many ways of defining these objects in set theory, and so to say R is a subset of C just doesn't cut it if you are going to be nit−picking.

Let's say I define the complex numbers C as an algebraically closed field of characteristic zero and cardinality the continuum with a distinguished automorphism conj(z) of degree two. Now I define the real numbers R as the subextension fixed by this automorphism, which I now dub "complex conjugation". Now I define an archimedian absolute value by |z| = sqrt(z conj(z)). Now C and R are topological fields under the topology defined by this, and conj is continuous. Now I define Q as the intersection of all subfields of C (or R.) Z is the ring of integers of Q. I've defined things so that, by definition, Q is a subfield of C. Of

course doing it your way I have a Q in C which is uniquely isomorphic
to the original Q which was constructed, and so forth blah blah.

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