Re: naive question from a non−mathematician

Source: http://sci.tech−archive.net/Archive/sci.math/2006−05/msg05677.html

- *From*: David C. Ullrich <ullrich@xxxxxxxxxxxxxxxx>
- *Date*: Tue, 30 May 2006 18:33:40 −0500

On 30 May 2006 13:11:38 −0700, "Gene Ward Smith"
<genewardsmith@xxxxxxxxx> wrote:

> David C. Ullrich wrote:
>
>> I'll confess that I don't know what a group of height one is.
>> But if we're talking about an order−complete group of height
>> one presumably it's an ordered group − I do know what that is.
>
> It's an ordered group G such that the only proper subgroup H with the
> property that if
> 0 <= a <= b and b is in H, then a is in H, is the trivial group. There
> isn't a nontrivial group
> inside of G defined by some bound.
>
>> How do you prove the _existence_ of a complete ordered group?
>
> If I have a group of height one, it's easy to see that it is
> archimedian.

The question was about completeness, not archimedeanness.

> If I take the additive group of the rationals and complete
> it, I get an archimedian−−ie, height one−−ordered group
> which is complete. Of course, all this does is constructs the additive
> group of the reals without defining any field properties.

Right. Of course taking the completion of the field Q is a lot
trickier than taking the completion of the group (Q.+).

So it turns out that in order to implement the construction of C and R that you had in mind we need to _start_ with the standard construction of R, but not notice that we've done so. And this is supposed to prove the important point that there are possible constructions other than the standard one.

Glad we finally got to the bottom of this.

> > Or, more to the point: How do you prove that there exists a complete ordered group by an argument that cannot be trivially modified to prove the existence of a complete ordered field?

> Don't think it can be done, but I'm open to suggestions.

> > Not going to bother suggesting that this is getting a little silly. Yes, when someone said something about _the_ foundations that was inappropriate, surely there are many ways to set things up. But possibly you should have had the example better prepared before starting to present it?

> Admittedly I was just winging it. But the two obvious ways to proceed had already been done, since they are obvious. That is, construct everything from the bottom up, N to Z to Q to R to C, or define R axiomatically and get everything else from that.

> > The idea that this is a _better_
> > way to construct the complex numbers and then the reals
> > seems a little far−fetched.

> Who said anything about "better"?

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David C. Ullrich
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