Re: Looking for a surjection or R^2

Source: http://sci.tech−archive.net/Archive/sci.math/2006−06/msg01403.html

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- *From*: The World Wide Wade <waderameyxiii@xxxxxxxxxxxxxxxxxxxx>
- *Date*: Thu, 08 Jun 2006 23:20:10 −0700

In article <4488be04$0$7763$7a628cd7@xxxxxxxxxxxxxxxxxxxxxx>,
Denis Feldmann <denis.feldmann.asupprimer@xxxxxxxxxxxxxxxx>
wrote:

> I am looking for a smooth surjection of R^2 onto itself, not bijective,
> with jacobian everywhere not zero. My best example so far is an
> holomorphic function (with the obvious isomorphism between C and R^2)
> like the antiderivative of exp(z^2) (which is surjective by Picard
> theorem), but I would like something simpler, more explicit, and where
> the proof of surjectivity uses only elementary calculations...

Let f be any smooth function on R such that f(y) = 0 if y <= 0,
f' <= 0 on [0,Pi], f' >= 0 on [Pi,2Pi], f(2Pi) < 0, and f(y) =
f(2Pi) for y >= 2Pi. Writing z = x + iy, define F(z) = e^z +
f(y). Then the Jacobian of F at x + iy is e^(2x) −
e^x*sin(y)*f'(y), which = 0 iff e^x = sin(y)*f'(y). The right
hand side is always <= 0, so the equation cannot be satisfied.
Therefore the Jacobian of F is never 0. F is surjective because
below the x−axis it maps onto C \ {0}, and on the line y = 2Pi it
is the function e^x + f(2Pi); because f(2Pi) < 0, this function
takes on the value 0.
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