Re: Looking for a surjection or R^2

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- *From*: "denis feldmann" <denis_feldmann@xxxxxxxx>
- *Date*: 10 Jun 2006 00:39:07 −0700

The World Wide Wade wrote:

> In article
> <waderameyxiii−45674F.23201008062006@xxxxxxxxxxxxxxxxxxxxxxxx>,
> The World Wide Wade <waderameyxiii@xxxxxxxxxxxxxxxxxxxx> wrote:
>
>> In article <4488be04$0$7763$7a628cd7@xxxxxxxxxxxxxxxxxxxxx>,
>> Denis Feldmann <denis.feldmann.asupprimer@xxxxxxxxxxxxxxxx>
>> wrote:
>>
>>> I am looking for a smooth surjection of R^2 onto itself, not
>>> bijective,
>>> with jacobian everywhere not zero. My best example so far is
>>> an
>>> holomorphic function (with the obvious isomorphism
>>> between C and R^2)
>>> like the antiderivative of exp(z^2) (which is surjective by
>>> Picard
>>> theorem), but I would like something simpler, more explicit,
>>> and where
>>> the proof of surjectivity uses only elementary calculations...
>>
>> Let f be any smooth function on R such that f(y) = 0 if y <= 0,
>> f' <= 0 on [0,Pi], f' >= 0 on [Pi,2Pi], f(2Pi) < 0, and f(y) =
>> f(2Pi) for y >= 2Pi. Writing z = x + iy, define F(z) = e^z +
>> f(y). Then the Jacobian of F at x + iy is e^(2x) −
>> e^x*sin(y)*f'(y), which = 0 iff e^x = sin(y)*f'(y). The right
>> hand side is always <= 0, so the equation cannot be satisfied.
>> Therefore the Jacobian of F is never 0. F is surjective because
>> below the x−axis it maps onto C \ {0},
>
> Here I should have said that below the x_axis, F(z) = e^z, where
> it maps many to one onto C \ {0}.

and on the line y = 2Pi it
is the function e^x + f(2Pi); because f(2Pi) < 0, this function
takes on the value 0.

And finally, F is not a bijection because of its behavior below
the x−axis.

Thanks a lot. Are you the author of this construction, or do you know
who he is?

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