

Re: Looking for a surjection or \mathbb{R}^2

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- *From:* The World Wide Wade <waderameyxiii@xxxxxxxxxxxxxxxxxxxxxxxx>
 - *Date:* Sat, 10 Jun 2006 10:21:16 -0700
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In article

<1149925147.272877.32240@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx>, "denis feldmann" <denis_feldmann@xxxxxxxx> wrote:

The World Wide Wade wrote:

In article

<waderameyxiii-45674F.23201008062006@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx>, The World Wide Wade <waderameyxiii@xxxxxxxxxxxxxxxxxxxxxxxx> wrote:

In article

<[4488be04\\$0\\$7763\\$7a628cd7@xxxxxxxxxxxxxxxxxxxxxxxx](mailto:4488be04$0$7763$7a628cd7@xxxxxxxxxxxxxxxxxxxxxxxx)>, Denis Feldmann <denis.feldmann.asupprimer@xxxxxxxxxxxxxxxx> wrote:

I am looking for a smooth surjection of \mathbb{R}^2 onto itself, not bijective, with jacobian everywhere not zero. My best example so far is an holomorphic function (with the obvious isomorphism between \mathbb{C} and \mathbb{R}^2) like the antiderivative of $\exp(z^2)$ (which is surjective by Picard theorem), but I would like something simpler, more explicit, and where the proof of surjectivity uses only elementary calculations...

Let f be any smooth function on \mathbb{R} such that $f(y) = 0$ if $y \leq 0$, $f' \leq 0$ on $[0, \pi]$, $f' \geq 0$ on $[\pi, 2\pi]$, $f(2\pi) < 0$, and $f(y) = f(2\pi)$ for $y \geq 2\pi$. Writing $z = x + iy$, define $F(z) = e^z + f(y)$. Then the Jacobian of F at $x + iy$ is $e^x \begin{pmatrix} 1 & i \\ -\sin(y) & f'(y) \end{pmatrix}$, which is 0 iff $e^x = \sin(y) \cdot f'(y)$. The right

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hand side is always ≤ 0 , so the equation cannot be satisfied.
Therefore the Jacobian of F is never 0. F is surjective
because
below the x -axis it maps onto $\mathbb{C} \setminus \{0\}$,

Here I should have said that below the x -axis, $F(z) = e^z$, where
it maps many to one onto $\mathbb{C} \setminus \{0\}$.

and on the line $y = 2\pi i$ it
is the function $e^x + f(2\pi i)$; because $f(2\pi i) < 0$, this function
takes on the value 0.

And finally, F is not a bijection because of its behavior below
the x -axis.

Thanks a lot. Are you the author of this construction

Yes.

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