Re: Looking for a surjection or R^2

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- *From*: Denis Feldmann <denis.feldmann.asupprimer@xxxxxxxxxxxxxxxx>
- *Date*: Sat, 10 Jun 2006 19:29:47 +0200

The World Wide Wade a écrit :

> In article <1149925147.272877.32240@xxxxxxxxxxxxxxxxxxxxxxxxxxxxx>,
> "denis feldmann" <denis_feldmann@xxxxxxxx> wrote:
>
>> The World Wide Wade wrote:
>>
>>> In article
>>> <waderameyxiii−45674F.23201008062006@xxxxxxxxxxxxxxxxxxxxxxxxx>,
>>> The World Wide Wade
>>> <waderameyxiii@xxxxxxxxxxxxxxxxxxxx> wrote:
>>>
>>>> In article
>>>> <4488be04$0$7763$7a628cd7@xxxxxxxxxxxxxxxxxxxxxx>,
>>>> Denis Feldmann
>>>> <denis.feldmann.asupprimer@xxxxxxxxxxxxxxxx>
>>>> wrote:
>>>>
>>>>> I am looking for a smooth
>>>>> surjection of R^2 onto itself,
>>>>> not bijective,
>>>>> with jacobian everywhere
>>>>> not zero. My best example
>>>>> so far is an
>>>>> holomorphic function (with
>>>>> the obvious isomorphism
>>>>> between C and R^2)
>>>>> like the antiderivative of
>>>>> exp(z^2) (which is
>>>>> surjective by Picard
>>>>> theorem), but I would like
>>>>> something simpler, more
>>>>> explicit, and where
>>>>> the proof of surjectivity uses
>>>>> only elementary
>>>>> calculations...

>>> Let f be any smooth function on R such that
>>> f(y) = 0 if y <= 0,
>>> f' <= 0 on [0,Pi], f' >= 0 on [Pi,2Pi], f(2Pi) <
>>> 0, and f(y) =
>>> f(2Pi) for y >= 2Pi. Writing z = x + iy,
>>> define F(z) = e^z +
>>> f(y). Then the Jacobian of F at x + iy is
>>> e^(2x) −
>>> e^x*sin(y)*f'(y), which = 0 iff e^x =
>>> sin(y)*f'(y). The right
>>> hand side is always <= 0, so the equation
>>> cannot be satisfied.
>>> Therefore the Jacobian of F is never 0. F is
>>> surjective because
>>> below the x−axis it maps onto C \ {0},
>>
>> Here I should have said that below the x_axis, F(z) = e^z,
>> where
>> it maps many to one onto C \ {0}.
>>
>>> and on the line y = 2Pi it
>>> is the function e^x + f(2Pi); because f(2Pi) <
>>> 0, this function
>>> takes on the value 0.
>>
>> And finally, F is not a bijection because of its behavior
>> below
>> the x−axis.
>
> Thanks a lot. Are you the author of this construction

Yes.

Thanks again. Of course, this has many merits, but one of those was to guide us towards a completely explicit (C−infinite) function, namely
F(x,y)= (sin(y)(e^x − 1/2), −cos(y) e^x) . I will try to give proper credits to it if we happen to publish it (in a course for French teachers)
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