Re: Looking for a surjection or R^2

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- *From*: lrudolph@xxxxxxxxx (Lee Rudolph)
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Denis Feldmann <denis.feldmann.asupprimer@xxxxxxxxxxxxxxxx> writes:

> I am looking for a smooth surjection of R^2 onto itself, not bijective, with jacobian everywhere not zero.

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> Thanks again. Of course, this has many merits, but one of those was to guide us towards a completely explicit (C−infinite) function

Here's the example with irregular branched coverings that I promised. *I* think it's already "completely explicit" as I'm about to give it; but further explicitation can be done (and will be sketched) at the expense of simplicity.

As before, identify R^2 with C, and let p(z)=z^3−3z. Then p is a 3−sheeted irregular branched cover of C over C, with critical points 1 and −1, and critical values −2 = p(1) = p(−2) and 2 = p(−1) = p(2). The preimage p^{−1}(R), call it G, is the union of R and two (real) parabolas, forming a figure approximately like this:

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in which the 2 doublepoints of G are the critical points of p. Note that C \ G the union of 6 open 2−cells, on each of which the restriction of p is a diffeomorphism onto either the upper or the lower half−plane; in fact the restriction of p to the closure of the open 2−cell is a homeomorphism onto the closed half−plane, which "straightens out" the right angle(s) on the boundary but is otherwise smooth. Let K be the closure of the "bottom" 2−cell, that is, the one that contains the negative imaginary axis, and U its complement. Then p has no critical points on U, p(U) = C, and p|U is not injective. Since U is manifestly diffeomorphic to C, we're done, by my standards. To bring the example up to what *may* be your more exacting standards, we need only write down a formula for a diffeomorphism of U to C. There's lots of ways to do that, which however I decline to do at the moment, thanks.

Lee Rudolph
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