

# Re: Looking for a surjection or $\mathbb{R}^2$

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Denis Feldmann <denis.feldmann.asupprimer@xxxxxxxxxxxxxxxxxxxx> writes:

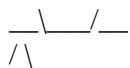
I am  
 looking for  
 a smooth  
 surjection  
 of  $\mathbb{R}^2$  onto  
 itself, not  
 bijective,  
 with  
 jacobian  
 everywhere  
 not zero.

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Thanks again. Of course, this has many merits, but one of those was to guide us towards a completely explicit ( $C$ -infinite) function

Here's the example with irregular branched coverings that I promised. \*I\* think it's already "completely explicit" as I'm about to give it; but further explicitation can be done (and will be sketched) at the expense of simplicity.

As before, identify  $\mathbb{R}^2$  with  $C$ , and let  $p(z)=z^3-3z$ . Then  $p$  is a 3-sheeted irregular branched cover of  $C$  over  $C$ , with critical points 1 and  $-1$ , and critical values  $-2 = p(1) = p(-2)$  and  $2 = p(-1) = p(2)$ . The preimage  $p^{-1}(R)$ , call it  $G$ , is the union of  $R$  and two (real) parabolas, forming a figure approximately like this:



in which the 2 doublepoints of  $G$  are the critical points of  $p$ . Note that  $C \setminus G$  the union of 6 open 2-cells, on each of which the restriction of  $p$  is a diffeomorphism onto either the upper or the lower half-plane; in fact the restriction of  $p$  to the closure of the open 2-cell is a

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homeomorphism onto the closed half-plane, which "straightens out" the right angle(s) on the boundary but is otherwise smooth. Let  $K$  be the closure of the "bottom" 2-cell, that is, the one that contains the negative imaginary axis, and  $U$  its complement. Then  $p$  has no critical points on  $U$ ,  $p(U) = \mathbb{C}$ , and  $p|_U$  is not injective. Since  $U$  is manifestly diffeomorphic to  $\mathbb{C}$ , we're done, by my standards. To bring the example up to what \*may\* be your more exacting standards, we need only write down a formula for a diffeomorphism of  $U$  to  $\mathbb{C}$ . There's lots of ways to do that, which however I decline to do at the moment, thanks.

Lee Rudolph

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